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New Jersey elementary school proficiency assessment (E.S.P.A) mathematics analysis: open-ended questions, scoring rubrics, developmental activities and manipulatives with scope of skills for grades K-4

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NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (E.S.P.A.) MATHEMATICS ANALYSIS : OPEN-ENDED QUESTIONS, SCORING RUBRICS, DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES WITH SCOPE OF SKILLS FOR GRADES K-4

by

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Abstract

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E.S.P.A. Mathematics Analysis : Open-ended Questions, Scoring Rubrics,Developmental Activities and Manipulatives With Scope of Skills For Grades K-4

2002

Dr. Louis Molinari

Elementary Education - Mathematics Emphasis

This document provides a resource which may be used by elementary teachers to gain understanding of the States Standards, the construction and purpose of E.S.P.A., the scoring of open-ended questions and the scope of development of fourth grade skills through the elementary grades. There are three sections of this document, which include the following : the "Macro Development and Open-ended Listing For Grade 4", the Mathematics Skill Development For E.S.P.A. : Grades K - 4" and the "Mathematics Manipulatives Listing". Each section provides important information pertaining to appropriate teaching to prepare students for E.S.P.A.

This document may be used by teachers in weekly planning, in monitoring the content of their curriculum guide or text book, and in assessing where their students are in the scope of elementary mathematics skills and where they need to be by the completion of the fourth grade. It may help to increase the implementation of quality mathematics education and as a result assist in the increase of student assessment scores on the E.S.P.A.

Mini Abstract

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E.S.P.A. Mathematics Analysis : Open-ended Questions, Scoring Rubrics, Developmental Activities and Manipulatives With Scope of Skills For Grades K-4

2002

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Elementary Education - Mathematics Emphasis

The purpose of this project is to create a resource document to be utilized by elementary teachers, in teaching mathematics and preparing students for the format and content of the New Jersey E.S.P.A.

The three sections of this document, including the following resource pieces : the "Macro Development and Open-ended Listing For Grade 4", the "Mathematics Skill Development For E.S.P.A.: Grades K-4" and the "Mathematics Manipulatives Listing". This document may assist in the implementation of quality mathemaitcs education and appropriate skill development which may assist in the increase of student learning and demonstrated proficiency on the New Jersey E.S.P.A.

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Thesis Chapter 1

State-mandated academic assessments in grades four, eight and eleven, have been administered to students, in the state of New Jersey, for the past four years. These test assessments were created to provide the state with information concerning student achievement in the four content areas, of the State Standards, which include the following: Science, Social Studies, Language Arts and Mathematics. The assessments are designed to gauge student learning and district program alignment with the State Content Standards in these areas. Baresic and Gilman (2001), in their article *How Does the Pendulum Swing on Standardized Testing?,* contend that such state mandated testing carries high stakes in that it becomes, in some states, the basis for grade level promotion, high school graduation and mandatory remediation services.

This is true, in the state of New Jersey, where students who test below the state proficiency level, in the areas of language arts or mathematics, are placed into remedial or Basic Skills classes, for those subjects, in the following school year. The test results impact greatly on district Basic Skills Programs by increasing the number of students in remedial programs and thereby increasing the cost of these programs to the individual districts.

In an effort to assist the districts in test preparation, the New Jersey State Department of Education releases volumes of data which explain the purpose, content and construction of the test. It also releases sample test questions, explanations of the basis of the questions, practice sample questions with scoring rubrics, and samples of original student responses from the state and the local district. However, most importantly it releases individual student scores and the percentage of students from each district who passed the proficiency requirement

for each of the tests.

The Elementary Student Proficiency Assessment, or E.S.P.A., is the fourth grade state mandated test. The E.S.P.A. mathematics scores in the Pennsauken School District, from the May 2001 testing, resulted in only 53% of the fourth grade students in the district passing the assessment. This did not meet the New Jersey minimum requirement of passing students in the district. Therefore, the Pennsauken Schools district objective for this school year is to increase the mathematics scores on the May 2002 E.S.P.A

The decreased assessment scores coupled with two other crucial changes make the increase of scores even more critical. First, the New Jersey State Department of Education has recently released information stating that all students taking the E.S.P.A., including regular education and classified special education students, both mainstreamed and fully contained, will be included in the districts percentages. To date, special education students scores were not grouped with those of regular education students. This could possibly result in a decrease of passing percentages. Second, this is a state monitoring year for the district and technically the district has already failed the monitoring due to the below standard percentage of passing students on the Mathematics portion of the E.S.P.A. However, since the district has already addressed the need for improvement, by establishing increasing mathematics scores as the district objective for this year, the state will continue with the monitoring procedure. It is possible for the district to obtain a provisional passing of the monitoring if the May 2002 scores reflect a substantial increase.

In an effort to address this deficit, the district has already committed to adopting a new mathematics series for the 2002-2003 school year. There have been various text books piloted by teams of classroom teachers and Basic Skills

teachers throughout the district. Those teachers involved in piloting comprise the text book selection committee. The committee will convene several times during the year in order to report on the success or failure of their pilot programs. Each text book company will give a presentation to the committee. The committee will evaluate and make recommendations as to the most appropriate text to address the needs of our students and the district.

The district is also committed to teacher education as an integral part of the improvement plan. The third and fourth grade, regular education and special education, teachers are being assisted by the Basic Skills teachers through district in-services. The Basic Skills mathematics teachers convened several afternoons in order to sift through the volumes of state released testing information. This was done in an effort to present the most pertinent information to the teachers. The goal was to redirect the teachers activities, in order to assist them in better utilizing their time and effort toward the main focus of E.S.P.A. This presentation was made to the teachers, in October 2001, through two in-service days wherein the Basic Skills teachers explained the construction of the assessment tool, analyzed test questions, reviewed scoring rubrics, and recommended specific strategies and implications for teaching. The teachers were overwhelmed by the extensive content of the assessment. The teachers expressed their serious concern as to the pressure they felt in completing the monumental task of preparing the fourth grade students for E.S.P.A. testing.

The presentation was helpful in alleviating some of the fears by giving the teachers some teaching strategies with more specific direction toward assisting their students for success on E.S.P.A It appeared that the teachers could benefit from a more organized guideline document. A resource was needed to organize the E.S.P.A. documents and to provide specific activities designed to develop

each of the mathematics content clusters and the twenty specific Macro Skills included in the assessment. The twenty mathematics Macro Skills, which are assessed on the E.S.P.A., are derived from the five mathematics Content Clusters. It was apparent that a document which would track the development of the mathematics skills which will be tested on the E.S.P.A. could be useful. The development of these skills begins in kindergarten and builds through the primary grades to grade four. Those teachers who are involved in teaching students in kindergarten through second grade should also be given the same direction and guidance as was given to the third and fourth grade. It is essential that all elementary teachers be informed of the mathematics skills included on the E.S.P.A. and are directed in specific teaching strategies designed to better develop these skills.

It is the purpose of this thesis document to create an elementary mathematics reference piece, with three sections, for use by regular and special education elementary school teachers. One section is specifically designed for use with fourth grade students who are taking the E.S.P.A. Another section is designed for use by all elementary teachers, to gain an understanding of the skills which are being assessed by the state testing. The last section is designed for use by all kindergarten through fourth grade teachers in order to assist them in the utilization of appropriate mathematics manipulatives during their mathematics instruction. The section for all elementary teachers follows the development of each of the Macro Skills from kindergarten through fourth grade. This content domain outline is intended to be used to follow the development of the Macro Skills, thereby allowing teachers to see where the skills they are teaching fit into the development of the student's overall mathematics knowledge.

The three sections of this thesis document may serve as a guidebook to

assist teachers in incorporating the necessary content and in utilizing appropriate activities and manipulatives to encourage skill development in order to prepare students for the assessment. It is hoped that teachers may incorporate these activities into their everyday planning, as an interwoven and integral part of their mathematics curriculum.

Included, within the sections of the thesis document, is an in-depth description of the Mathematics Content of the E.S.P.A., which is based on the N.C.T.M. Standards (1989). This provides a description of the New Jersey E.S.P.A Mathematics Content Clusters and Macro Skills. The five clusters include the following :

1.Number Sense, Operations and Properties

2. Measurement

3. Spatial Sense and Geometry

4. Data Analysis, Probability

5.Discrete Mathematics and Patterns and Algebra.

The Macros are the twenty main sub-skills in mathematics which are being assessed on the E.S.P.A. The Macro sub-skills link directly to the Content Clusters and to the sixteen New Jersey Mathematics Standards.

Also included in this thesis document, in Chapter 3, is an analysis of the state generic scoring rubric. Mabry (1999), in her article *Writing to the Rubric : Lingering Effects of Traditional Standardized Testing on Direct Writing Assessment*, contends that rubric scoring is crucial in current implementations of academic standards and that it is essential to understand the workings of the rubric and it's application to student's responses. She explains that it is the strict compliance to specific rubric criteria, being used by the scorers, which governs the student's score and the accuracy of the assessment.

To date, this type of tool for teacher use has not been created. It is proposed that teachers may use sections of this thesis document to track the development of each of the Macro Skill areas throughout the school year. In the past, teachers may have followed the textbook from cover to cover or used the curriculum guide in planning lessons, saving certain skills to be taught after the test. It is important for teachers to know ahead of time which skills are necessary to cover prior to the test. It is proposed that having an easily accessible piece which teachers may use to chart their progress in covering the necessary skills may result in more efficient planning and more effective teaching. This will also assist teachers in incorporating the necessary content skills into their normal scope and sequence of mathematics teaching throughout the year.

The first two sections of this thesis document include specific objectives, activities and student outcomes, designed to promote an increase in teacher understanding of the Content Clusters and the Macro Skills being assessed on the E.S.P.A. It is hoped that this will better explain to all elementary teachers the importance of their contribution to the student's mathematical knowledge.

The specific constructivist activities included in these two sections are designed to foster the development of the conceptual and procedural knowledge incorporated into the Macro Skills. Included are hands-on, manipulative based activities which engage the students in problem solving activities creating the link between the content and it's application. Many mathematics manipulatives are listed as being useful in this content and skill development.

The third section presents a list of many of the manipulatives which are named throughout sections one and two in the teaching strategies and constructivist activities designed to enhance teaching of all mathematics areas included on the E.S.P.A. Many of the manipulatives listed are commercial

products. Others are common game pieces. Included in this list are the many uses and appropriate grade levels with which each of he manipulatives may used. It is essential for student understanding, that the building of mathematics content be presented through the use of quality constructivist activities. The use of specific manipulatives, which build through the grades, also assists in establishing the interconnectedness of mathematics skills and content in students. Schwartz (2000), in his article, *Axing Math Anxiety*, suggests many ways in which teachers can promote a feeling of confidence in their students as they prepare them to undertake these assessments. The suggested strategies develop mathematics content skills, provide hands-on activities and, as a result, increase test scores.

Stipek, Given and Salmon (1998), in their article, *Can a Teacher Intervention Improve Classroom Practices and Student Motivation in Mathematics?,* helped to institute the change from "drill and test" and "paper and pencil" mathematics to provide a more meaningful real-life mathematics experience for the students. This change in the approach to education was in response to the call for reform in mathematics teaching as initiated by the Curriculum and Evaluation Standards for School Mathematics published by the National Council of Teachers of Mathematics (1989). The N.C.T.M. Standards served to guide curriculum reform, initiate text book revisions, and also formed the basis for the New Jersey Frameworks and state mandated testing.

Burns (1994), in her article *Arithmetic : the Last Holdout*, describes the message of these Standards as being the following : to teach the students to solve problems, to reason, to communicate, to value mathematics and to become confident in their own ability to do mathematics. These activities and appropriate test practice questions will address the standards while developing student understanding of content skills and their application of them. It is hoped that all

of these may be contributing factors which serve to ultimately increase student test scores and achievement levels on standardized assessments.

It is the purpose of this project to direct elementary teachers to the types of activities which will assist in developing their student's mathematics learning and understanding and thereby better prepare them for the E.S.P.A. It outlines skill development and suggests manipulatives to use in constructivist activities. The activities are designed to utilize manipulatives to promote a better understanding of mathematics concepts in students.

Elementary teachers have an immense amount of content to teach during the course of the day. Each year new subjects are added to their day with none being removed. Along with the four major subject areas of language arts, mathematics, science and social studies, they are also expected to teach many non-academic topics such as health education and character education. In the present year, the teachers are being challenged to greatly increase their student's mathematics test scores.

It may be that the activities and information created as a part of this study will be used by teachers to direct their teaching in a more productive manner, in order to assist them in their task of better preparing their students to meet the challenges of E.S.P.A. It is planned to disseminate this information to all kindergarten through fourth grade regular education, special education, Basic Skills and Resource Room teachers in Pennsauken, through faculty meetings and district in-service programs. It is also proposed to use the format and some of the documents to create a workshop for teachers at the New Jersey Association of Federal Program Administrators Spring Conference, April 25, 2002.

An E.S.P.A. Mathematics Glossary of Terms

1.Number Sense

Comparing Numbers (>, <, +)

Fractions, Decimals and Percentages (less than whole numbers, breaking wholes into parts)

Place Value

Negative Numbers

2.Operations

Numerical Operations using Whole numbers, Fractions, Decimals and Percents

Properties of Numbers

3.Measurement

Not Limited to Linear- includes units of measure - (i.e. volume, weight, etc.)

Elapsed Time (time in minutes to days)

Maps and Scales

Temperature

Perimeter and Area of a Shape

Standard and Metric Units

4. Spatial Sense and Geometry

Ability to "feel shapes in space"

Real-life Skill Development

Developed by manipulating Pattern Blocks, Pentominoes. etc.

5. Probability

The Chance(s) that something will happen

Representation by Fractions, Decimals and Percents

Involves Random Number Generators (i.e. dice, spinners)

6. Discrete Mathematics

Combinations and Permutations

Development of the concept that there are more than one, often many possibilities

7. Patterns

Various Types of Patterns (i.e.. number, color, repeating, growing, etc.)

Growing Patterns as a type of Calculus

8. Algebra

Functions : "What's My Rule ?"

Input/output Boxes

Missing Addends

9. Content Clusters

One of the five main mathematics skills included in the state standards

Are grouped into five clusters as follows:

1.Number Sense, Operations, and Properties

2. Measurement

3. Spatial Sense and Geometry

4. Data Analysis, Probability, and Discrete Mathematics

5. Patterns and Algebra

10. Macro Skill

One of twenty main Mathematics Skills included in the State Standards

Are incorporated into one of the five Content Clusters

11. Power Base Standards

Mathematics Standards which support the knowledge and problem solving standards

Skills which are interwoven into the Content Clusters and include the following: Reasoning, Connections, Communication, Problem Solving, Tools and Technology, Estimation and Higher Level Thinking

12. Scoring Reader

Professional reader/ scorer for the open-ended section of the E.S.P.A.

Hired by the State of New Jersey

Does not know the student whose test is being scored

13. Mathematics Manipulatives

Specific materials used in math teaching

May be commercially produced or common game pieces

Assist in student understanding

Integral part of the Hands-on approach to teaching and learning mathematics

Thesis Chapter 2

It is the purpose of this thesis project to provide a document to be utilized, by elementary classroom teachers, devoted to the mathematics section of the New Jersey Elementary Proficiency Assessment, or E.S.P.A. The project attempts to present an explanation of the test including the following components : skill content of the test, analysis of test questions, analysis of scoring rubrics, specific skill development activities and manipulatives with implications for future teaching.

This literature review will explore the purpose of open-ended questioning and performance based assessment including the use of a scoring rubric. In addition, the importance of the use of constructivist, hands-on, activities and the role they play in the development of mathematics concepts and understanding will be investigated. Included is a historical background and an explanation of the purpose of state-mandated testing.

Perrault (2000), interviewed teachers concerning their perceptions of the effects of state-mandated testing. Teachers reported feeling pressure to improve test scores from the first day of school. They also were directed to limit their curriculum to only the content included on the test. They felt that the purpose of education had been changed to include only the goal of passing the test at the exclusion of such goals as encouraging the joy of learning. In low-achieving schools, the principals directed teachers to focus on a step-by-step approach to improving student test taking skills. In high-achieving schools, principals encouraged enrichment activities of students as long as the scores remained high. "These conversations with teachers indicated that the battle to control the

ideology of school reform is being dominated by forces that lead to standardization of curriculum and instruction and to holding teachers accountable for students' mastery of basic skills." (Perrault 2000)

By limiting curriculum content and putting added pressure on the classroom teachers a benefit can not be expected for students.. "To the extent that educators are motivated to produce high test scores, such tests can have the effect of suppressing efforts to expand higher order skill teaching." (Resnick, 1987) It is through an understanding of the purpose and content of the mathematics standards and the test format, and through the use of specific developmental activities, that scores may be increased.

It is important for teachers to prepare students to be successful on the E.S.P.A. However, teaching to the test does not develop the fundamental knowledge and concepts which are necessary for students learning in order to promote understanding. Behr, Lesh and Post (1983) contend that it is real life experiences, manipulative models and the students translation of them which contribute to student learning and understanding. " Part of what we mean when we say that a student understands an idea like 1/3 is that: (1) he or she can recognize the idea embedded in a variety of qualitatively different representational systems, (2) he or she can flexibly manipulate the idea within given representational systems, and (3) he or she can accurately translate the idea from one system to another." (Behr, et al., 1983) It is the content of mathematics which needs to be taught to students in a meaningful way.

This project will attempt to assemble the mathematics standards and information provided by the state regarding E.S.P.A. into a document which will provide teachers with organization, specific developmental activities to promote understanding and greater direction for their teaching. It is hoped that this project

will promote the teaching of the necessary skills of creative problem solving, critical thinking and life-long learning skills. Without a new direction, "...teachers will forgo the kind of honest inquiry and exploration of skills needed in the real world, to make sure students are ready to take tests or address standards." (Ohler, 2001)

The National Council of Teachers of Mathematics Curriculum and Standards for School Mathematics (1989) created a new wave of thinking in regard to what students should be taught in mathematics and how teaching and assessment should take place. The impact of this reform caused educators from other subject areas to develop their own content standards. As a result, every state has adopted their own content standards based on those of the national organizations. There are three types of standards which include content standards, performance standards and delivery standards.

The content standards are a list of skills and concepts that students should learn ; they tell what should be included in an assessment in order to identify student proficiency. Performance assessment deals more specifically with what students should be able to do to demonstrate their proficiency. The delivery standards comprise the resources and quality of education by which students are being taught. These three types of standards have been combined in the New Jersey state standards. "Some state standards appear to be an amalgam, statements of what it was hoped students might learn (i.e., content standards) interlaced with statements of how well students were expected to perform (i.e., performance standards.)" (Mabry 1999)

With the emphasis being placed on student proficiency on the E.S.P.A. and other state assessments, it has become increasingly important to develop reliability in scoring. The scoring rubric, which is used in the open-ended

response section in mathematics, is designed to evaluate both content and performance standards. It is also necessary that scorers be guided in their judgment of students written responses. "Rubrics tend to improve interrater reliability, the likelihood that different raters will award similar scorers. Not, coincidentally, there is a growing presumption that good performance assessment requires a rubric." (Mabry 1999) The open-ended response questions require students to construct their response in a specific manner in order to demonstrate both their knowledge and application of skills. Scoring rubrics have been developed to assist the state scorers in handling the large volume of responses on the standards-based performance section of the E.S.P.A. assessment and in safeguarding reliability.

The N.C.T.M. Standards also caused revisions in mathematics instruction which impact on preparation for E.S.P.A. The development of understanding of mathematics concepts and application of the standards is the core of E.S.P.A. and its educational purpose. It is necessary to continue to teach computation and basic facts, while incorporating them into a problem solving curriculum which fosters mathematics thinking. "The overall message has been a consistent one : teach the children to solve problems, reason, communicate, value mathematics and become confident in their ability to do mathematics. Teaching for understanding is in ; learning rote skills is out." (Burns, 1994). Development of specific problem solving activities in skill content areas are the essential components of the Macros, which are tested in the open-ended responses, being derived from the New Jersey State Standards for mathematics education.

The State Standards are based on the N.C.T.M. Standards which call for specific constructivist activities designed to develop student understanding and to build the link needed to achieve appropriate application of those skills. The

constructivist approach in teaching is one that provides hands-on activities which encourages students to think, connect and understand in order to learn. "Schools can become settings in which students are encouraged to develop hypothesis, to test out their own and others ideas, to make connections among content areas, to explore issues and problems of personal relevance and to work cooperatively with peers and adults in the pursuit of understanding, and toward the disposition to be life long learners." (Brooks and Brooks 1993) Through the use of constructivist activities, the student develops understanding of a specific concept and is then able to apply that concept in a problem solving setting.

As a result of the N.C.T.M. Standards, the use of manipulatives to assist in content development has become a core element of the mathematics curriculum. It is also essential to include student reflection as an integral part of the thinking and writing component in mathematics development. A diagnostic assessment of E.S.P.A. as a testing tool, including a skills analysis gives vital information on the type of manipulative activities needed for development of skills being assessed. A skills analysis assists teachers in providing the appropriate concept development and in determining the need for reinforcement or alternate examples. The actual test kit provides students with specific manipulatives, to use on the assessment, with which they must become familiar. "The relevant application of manipulatives to real-world as well as classroom situations helps students visualize and develop problem solving strategies."(Moch, 2001) It is thought that teachers time is better put to good use by using manipulatives for concept development in the first place than in using them in re-teaching concepts missed by students.

It has been common practice for mathematics remediation to include manipulative development of a particular concept. In some states, as in New

Jersey, the state-mandated testing has been used as an indicator of student placement into remedial classes. Often, low test results determine a students grade retention, or may even possibly hinder high school graduation. "Advocates consider these tests to be a quick, relatively inexpensive way to determine student growth and achievement. " (Baresic, 2001) When test scores indicate the need for remedial courses, the remediation is often solely an effort to get scores up on the test.

The remediation needs to develop test preparation skills along with further development of conceptual understanding, which were missed by the student, through the use of specific manipulative activities. Incorporating the manipulative approach in mathematics, Ediger (1998) presents an educational model which utilizes varied manipulatives including the following : attribute blocks, beads, pattern blocks, geometric shapes, and place value building blocks. In his article he further discusses how each of these items can be used to introduce or develop a basic mathematics concepts.

It is thought that through the use of specific manipulative activities that students can truly learn and apply mathematics concepts. "Authentic assessment is intricately linked with authentic teaching, which consists of modeling, coaching, and organizing the thinking of the students, and creating the conditions for education to happen." (Campbell, 2000) It is imperative that teachers understand the connection between the constructivist activities designed to develop concepts and the learning that results from these experiences. Their approach to teaching will impact their students learning. It is hoped that this thesis project will replace "teaching to the test" with a more authentic approach to teaching and education.

The goal of authentic assessment of student performance and rubric

assessment has been attempted in the open-ended section of the E.S.P.A. Through the use of real-life activities, students are able to demonstrate their problem solving capabilities. The Macro content presented in the open-ended setting affords the student the opportunity to demonstrate their own personal learning style and individuality of understanding. Students need to acquire problem solving not only for the purpose of being successful in school, but also, in order to live in the more complex world of the future. "For example, our children will need to know how to frame problems for themselves, how to formulate plans to address them, how to assess multiple outcomes, how to consider relationships, how to deal with ambiguity, and how to shift purposes in light of new information." (Eisner, 1999) Open-ended responses with rubric assessment helps to de-standardize the standardized assessment tool. It offers us an arena in which we may secure information about learning that can help to improve the curriculum and teaching methods. "In short, it affords us an opportunity to use evaluation formatively and to treat assessment as an educational medium." (Eisner, 1999) By analyzing student responses, insights are gained into student thinking which may be used to develop prescriptive teaching for the future.

As the emphasis is placed so heavily on test score improvement, it remains the job of educators to continue to teach the curriculum and not to isolate test skills or content to the exclusion of other content. It is hoped, that through the use of this project, teachers will emphasize the development of math concepts included on the E.S.P.A. within the context of the regular mathematics curriculum and through the course of the entire school year. "The most obvious consequence of an increased emphasis on assessment is that... there is little strategic theory fitting pedagogy to assessment ; thus fewer teachers know how to respond to poor

student performance, other than to try harder. Therefore, over emphasis on assessment erodes confidence in legitimate teaching competence." (Stake, 1999) It is recognized that classroom teachers are putting much effort into preparing students for E.S.P.A. It appears that at the time that they are feeling the most pressure, they may be feeling the least competent in completing the task. A goal of this project is to provide teachers with organized information and a new direction for their teaching which will in turn better prepare their students for E.S.P.A.

The E.S.P.A. has greatly changed the manner in which mathematics is being taught and the content which is being included in the mathematics curriculum. A similar situation occurred, in science education, in the state of New York.. Mathison in her article, *Implementing Curriculum Change Through State-mandated Testing : Ethical Issues* (1991), describes how state-mandated testing is developed in an effort to control what is happening in public schools. She contends that it is the testing tool itself which will promote change in the way a specific subject is taught. The tests are, in her opinion, a strategy by which a state department of education can bring about its reform goals. The testing evaluation dictates the certain way in which the subject is subsequently taught.

The purpose of Mathison's article is to evaluate the validity of this type of policy implementation. She describes the Bureau of Science in the New York State Education Department and its attempt to implement a science syllabus in 1985. The syllabus described the goals of science education without specifying textbooks, activities or teaching strategies. It did not have much of an effect on science education in the state. As a result, the Elementary Science Program Evaluation Test, E.S.P.E.T. , was developed and implemented for fourth grade students. The main goal of the Bureau of Science was to increase the amount of

science being taught in elementary schools and to implement a hands-on, inquiry approach to science teaching.

After the instituting of the E.S.P.E.T., New York fourth grade teachers were surveyed in order to assess the impact of the test. Results indicated that 68% of the teachers felt the test focused on what should be taught in science, 69% reported that they were using more hands on activities, and 50% reported that they were now spending more time teaching science in their classrooms. From this data, it can be concluded that the E.S.P.E.T. did result in more science being taught with more hands-on activities. However, there were also some negative outcomes of the test. The survey also reported that 59% of teachers felt they would be evaluated by their students' test results, that 78% admitted to feeling pressure to have students do well on the test and 50% thought the E.S.P.E.T. was not and accurate reflection of their schools' science curriculum.

In general, most teachers perceive the changes brought about by the E.S.P.E.T. as being for the better. They report personally enjoying the hands-on approach to teaching science. However, Mathison contends that when the teachers realize that the test has become their curriculum that they will feel the effects of having relinquished their professional judgment. The E.S.P.E.T. demands that all teachers teach science in a certain manner. Mathison contends that this was not done in an ethical manner. "...two important related ethical concerns are missing from the N.Y.S.E.D.'s utilitarian view : the distribution of consequences among constituents and an ethic of caring." (Mathison, 1991) The interconnected relationship of students, teachers and administrators was overlooked, thereby sending an uncaring message, as all were in some way negatively effected as a result of the test.

There appear to be many historical similarities between the

implementation of the New York Sate E.S.P.E.T., in science, and the New Jersey E.S.P.A., in mathematics. In many respects, both tests are driving the curriculum in a particular direction, leaving classroom teachers feeling pressured and frustrated.

This project was created to assist in alleviating some of the pressure felt by teachers as a result of low scores on the mathematics section of E.S.P.A. It provides elementary teachers with information on specific mathematics content development, the use of manipulatives in constructivist activities, and a guide for rubric scoring to use in their mathematics teaching and in preparing students for E.S.P.A. The goals of this thesis project are to assist teachers and students in mathematics teaching and learning, increase scores on future state testing and hopefully build a better understanding of mathematics concept development.

Thesis Chapter 3

There are three components included in this thesis document. All three sections deal with the topic of mathematics education for kindergarten through fourth grade students. Each section is designed to highlight an integral part of the content which is being assessed on the fourth grade New Jersey assessment piece, or E.S.P.A. This thesis document is intended to be used as a resource for elementary teachers, to inform them of the content and appropriate skill development which is necessary for E.S.P.A. preparation and mathematics education.

The first section of this thesis project is the "Mathematics Macro Development and Open-ended Questions For Grade 4". It is designed for specific use by fourth grade teachers who are preparing students to take the E.S.P.A. In it are contained samples from each of the twenty Macro Skills and includes the following information for each sample : a description of the Macro Skill content with the coordinating State Standard code numbers, the knowledge and problem solving skills which are included in the Macro Skill, a listing of appropriate constructivist activities and manipulatives which may be used in that skill development, and an accompanying open-ended question and specific rubric designed for that particular Macro Skill. The student activities, open-ended questions and scoring rubrics may be from the *Directory of Test Specifications and Samples Items for the E.S.P.A. in Mathematics* (1998) published by the state of New Jersey or may be created by this researcher.

The mathematics skills which are being tested on the E.S.P.A. are based on the New Jersey State Mathematics Standards. These standards were adopted

by the state and are based on the N.C.T.M. Standards (1989). The state test writers used the national standards to create the content clusters for mathematics. The content clusters group the 16 standards into five major areas of study. Each of the five Content Clusters create and encompass a designated area in the course of study in the mathematics curriculum. The five Content Clusters include the following :

1. Number Sense : numeration, place value, whole numbers, fractions, decimals, money, calculator use, operations and properties of numbers
2. Measurement : standard and nonstandard measures of length, distance, weight, capacity, area volume, time and temperature
3. Spatial Sense and Geometry : geometric terms, shapes, angles, coordinate paths, maps, tables and grids
4. Data Analysis, Probability and Descrete Math : determining probability, predicting outcomes, and collecting, organizing, analyzing and interpreting data, describing algorithms, and describing combinations and permutations
5. Patterns and Algebra : Describing and verifying generalized patterns and relationships

These Content Clusters include all of the process standards, which are supported by the Power Base Standards. The Power Base Standards include Reasoning, Making Connections, Communication, Problem Solving, Using Tools and Technology Estimating and Using Higher Level Thinking Skills. These standards must be integrated into all the content standards through the use of technology activities including investigating, observing, building and writing in the area of mathematics.

The Content Clusters are designed to develop both conceptual and

procedural knowledge of the standards and also to develop problem solving skills in all areas. It is the quality of the interweaving, of the Content Clusters and the Power Base Standards, which builds the knowledge and skills needed to understand and apply the mathematical concepts being taught in our curriculums and tested on the E.S.P.A. In order to properly develop these concepts, it is necessary to engage students in constructivist activities which incorporate these areas in order to develop the necessary problem solving skills and knowledge identified in the Mathematics Standards.

The five Content Clusters are constructed of twenty Macro skill areas, which distinctly identify the skills included in the Mathematics Standards. The Macro Skills are the basis for every open-ended questions. By analyzing each open-ended question, it is possible to identify exactly which Content Cluster and Macro Skill is being assessed. Not every one of the twenty Macro Skills will be tested on any one specific E.S.P.A. assessment, however, it may be beneficial to examine each type in order to be able to better prepare students for each of the various types of questions they may encounter.

The open-ended questions on the E.S.P.A. are assessed through the use of a scoring rubric. The generic scoring rubric is based on a 0 - 3 point response system, with 3 being the highest score possible. It helps to ensure that students are fairly scored, as they are being assessed in their demonstration of their knowledge and skills on all test questions. The generic scoring reflects the student's abilities in the areas of understanding of essential mathematics concepts and in the execution of necessary procedures. The state's scoring reader is viewing the student's ability to incorporate their knowledge, problem solving skills and mathematics content into their written response. The scoring reader also assesses the student's written response in regard to relevance and to the

quality of the student's explanation of the solution. The generic scoring is based on the following criteria :

E.S.P.A. Math Scoring 3 : Complete understanding of essential math concepts, complete execution of procedures, few minor errors, if any, in relevance of response, clear and effective explanation with details of how the problem was solved. (The scoring reader does not need to infer how or why the scoring decision was made.)

E.S.P.A. Math Scoring 2 : Nearly complete understanding of essential math concepts, nearly complete execution of procedure, minor errors in relevance of response and may not be totally clear in detailing of the explanation of how the problem was solved. (The scoring reader may have to make some inferences.)

E.S.P.A. Math Scoring 1 : Limited understanding of essential math concepts, incomplete and/or contains major errors in execution of procedures, probably contains major errors in relevance of response, incomplete explanation with regard to how the problem was solved (The scoring reader may question how and why student decisions were made.)

E.S.P.A. Math Scoring 0 : Insufficient understanding of essential math concepts, Major errors in execution of procedures, major errors in relevance, may be no problem solving explanation given (Scoring reader may not be able to understand the explanation or why decisions were made.)

Through the explanation of the Content Clusters and the open-ended questions with rubric scoring, in this section of the project, fourth grade teachers may gain a better understanding of the construction of the mathematics portion of the E.S.P.A. They may utilize the prescribed activities and manipulatives and critically examine the open-ended questions and accompanying rubric scoring in

order to incorporate them in preparation with their students.

The second section of this thesis project is the "Mathematics Skill Development For E.S.P.A.- Grades K-4". It is based on the content domain outline of E.S.P.A. mathematics skills presented in the Harcourt (2002) text book edition. This section of the thesis describes the twenty Macro Skills being tested on the assessment. It tracks the introduction and development of each of the Macro Skills from kindergarten through fourth grade. It also details appropriate activities and mathematics manipualtives which may be used to present and extend these concepts to the students.

The mathematics skills which are being tested on the E.S.P.A., in grade four, are largely being developed from the time the child enters kindergarten and formal mathematics education begins. The five Content Clusters and the twenty Macro Skills are not only the basis for the fourth grade mathematics curriculum. These skills are being built in a scaffolding manner, while expanding the student's depth of understanding beginning in kindergarten and continuing through the primary grades. All of the grades leading to fourth grade share an equal part in the student's mathematics development. Therefore, it is felt that all elementary teachers may benefit from the availability of a scope of mathematics skills contained in the macro skills being assessed on the E.S.P.A.

The second section provides elementary teachers with a tool which they may utilize in order to identify which skills they are teaching that link directly to E.S.P.A. preparation. Included are student outcomes, specific activities and appropriate manipulatives which may be used in skill development. It is presented on a spread sheet in order for the information to be easily accessible to teachers of any elementary grade level

The third section of this thesis project is the "Mathematics Manipulatives

Listing". This section of the project is a listing of many mathematics manipulatives which have been utilized by this researcher, in the skill development of the State Mathematics Standards. Included in this section is a brief description of the manipulative, with which grades it is appropriate to use, and a list of which skills it may be used to develop. These manipulatives have been recommended in the first two sections, of this project, for specific use in Macro Skill development. The teacher may use the "Mathematics Manipulative Listing" to identify unfamiliar manipulatives or to find a new use for a familiar item.

Manipulatives have become an integral part in the development of mathematics skills. Many manipulatives may have multiple purposes or may be utilized in many grade levels. It is essential for the teacher to identify which skill each manipulative will be used to develop. It is also extremely important to identify which manipulative is appropriate for a specific grade level. Since there are so many valuable manipulatives, which may be used as mathematics tools in promoting student understanding of mathematics concepts, the teacher has many choices when creating lesson plans.

It is may be that elementary teachers may utilize this thesis document to assist in planning and preparing appropriate lessons designed to further enhance student mathematics understanding of the content contained in the State Standards. Through this effort, it is hoped that student overall preparation for E.S.P.A. and development of mathematics knowledge may be increased, resulting in an increase in student proficiency on the mathematics portion of E.S.P.A.

Thesis Chapter 4 :

The Thesis Project

New Jersey Elementary School
PROFICIENCY ASSSESSMENT (ESPA)

MATHEMATICS
MACRO SKILL DEVELOPMENT: OPEN-ENDED QUESTIONS, SCORING RUBRICS, ACTIVITIVES AND MANIPULATIVES

GRADE 4

Joyce Scholz
Pennsauken Public Schools

INTRODUCTION

This resource booklet is designed to assist fourth grade teachers in their mathematical content development and preparation of students taking ESPA. The project included a listing of the mathematics Content Clusters and Macro Skills included on the assessment.

The booklet contains a description of each of the five Content Clusters with accompanying mathematics Standard code numbers. It describes the knowledge, and problem solving skills contained in each of the twenty Macro Skills which are included in the Content Clusters. It also includes developmental activities, open-ended questions and scoring rubrics for each Macro Skill to be used for classroom practice with students.

The goal of this resource is to present both the content and format of the open-ended section of ESPA. It may serve as a useful tool in fourth grade preparation for ESPA.

CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

Informed By Content Standards: 4.6, 4.8

Informed By Cross-Content Standards:4.1, 4.2, 4.3, 4.4, 4.5, 4.10, 4.16 Power Base

MACRO (A): (4.6.1, 4.6.5, 4.6.9)

Demonstrate meaning for whole numbers, negative integers, commonly used fractions, and decimals using physical materials, technology, and real-life experiences.

KNOWLEDGE: (4.6.1, 4.6.5, 4.6.9)

The student should have a conceptual understanding of:

1. Whole Number
2. Negative Integers
3. Commonly used fractions (halves, thirds, fourths, sixths, eighths, tenths)
4. Decimals (tenths, hundredths, values greater than one
5. Number line with integers, commonly used fractions, and decimals

PROBLEM-SOLVING SKILLS: (4.6.1, 4.6.5, 4.6.9, 4.8.1, 4.8.7)

In problem settings, using abilities that comprise the power base, the student should be able to:

6. Use whole numbers, negative integers, commonly used fractions, and decimals
    a. Whole numbers (up to and including seven digits)
    b. negative integers using number line and/or thermometer
    c. commonly used fractions
    d. decimals (up to and including hundredths place)

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build decimals using place value 100 flats, tens, ones

♦ Create diagram of decimals using graph paper

3. Match decimal diagrams to numerical representation
4. Sort decimals into three groups and explain how decision was made. Use three cups and label "near 0", "about ½", "near 1"
5. Use decimal models to locate points on 0-1 number line
6. Build fractions using pattern blocks and Cuisinaire rods
7. Trace pieces to model fractions of a shape
8. Use fraction circles to model parts of a whole
9. Use fractions models to locate points on a number line between 0 and 1
10. Use centimeter graph paper to diagram models

OPEN-ENDED QUESTION:

Marci and her brother, Mark, ordered a pizza. The sliced pizza is shown below. Mark ate 3 slices of the pizza and Marci ate 2 slices.

- How much of the pizza is left over?
- Explain how you got your answer.

(Provide Fractions Circle diagram cut in twelfths.)

SCORING RUBRIC:

3 points	The student determines that Mark ate 3/8 of the pizza and that 2/8 is left, labeling the diagram. The explanation is clear and logical in conveying knowledge of naming fractions.
2 points	The student determines that Mark ate 3/8 of the pizza and that 2/8 is left and provides a vague or incomplete explanation.
Or	The student, due to a minor error, gives an incorrect answer but gives a clear and logical explanation of how the problem was solved.
1 point	The student attempts to name the fractional parts, showing some understanding, however, the work shows errors in procedures or explanations are incomplete.
0 points	The response shows insufficient understanding of the problem's mathematical concepts.

MACRO (B): (4.6.2, 4.6.3)
Show understanding of place value concepts and numeration using counting, grouping, and pattern identification.

KNOWLEDGE: (4.6.2, 4.6.3)
The student should have a conceptual understanding of:

1. Counting/place value relationship
2. Grouping and place value
3. Expanded notation and numeration
4. Number patterns
    a. multiplication by 10, 100, 1000… (powers of ten)
    b. ten-to-ten relationship of adjacent place values
    c. periods
5. Rounding and place value

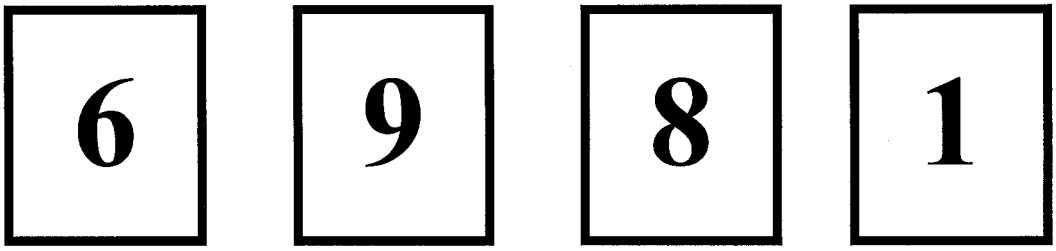
PROBLEM-SOLVING SKILLS (4.6.2, 4.6.3)
In problem settings, using abilities that comprise the power base, the student should be able to:

6. Use place value concepts and numeration
    a. use skills in counting on and counting groups
    b. use models to demonstrate regrouping
    c. identify the value of a digit in a given number
    d. write a number in expanded form or standard form
    e. round a whole number to the nearest ten, hundred, or thousand

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES: Counting/place value/grouping

1. Build numbers using place value blocks
2. Create diagrams of place value blocks on centimeter graph paper
3. Use 0-9 digit cards to make largest/smallest numeral
4. Roll dice and arrange numerals tossed to create largest 6-digit number
5. Record numbers in place value chart

OPEN-ENDED QUESTION:
You have the following four number cards:



- What is the smallest four digit number you can make? _____

- What is the largest four digit number you can make? _____
  (Show your work and clearly explain your answer.)

SCORING RUBRIC:

3 points	The student determines 1,689 to be the smallest number and 9,861 to be the largest number you can make. The explanation is clear and logical.
2 points	The student determines the correct answer for both and provides a vague or incomplete explanation.
Or	The student, due to a minor error gives and incorrect part, but gives a clear and logical explanation.
1 point	The student attempts to determine the 2 numbers, showing some understanding, however, the work shows errors in procedure or explanations are incomplete.
0 points	The response shows insufficient mathematical concepts.

MACRO (C): (4.6.4, 4.6.7, 4.6.8)
Compare and order numbers, commonly used fractions, and decimals.

KNOWLEDGE: (4.6.4, 4.6.7, 4.6.8)
The student should have a conceptual understanding of:

1. Models for whole numbers, fractions, and decimals
    a. base ten blocks
    b. fraction pieces (bars, circles, etc.)
    c. money
    d. graph paper
2. Models for place value
    a. base blocks
    b. place value charts
3. Symbolic place value

The students should be able to:

4. Use symbols: <, >, =

PROBLEM-SOLVING SKILLS: (4.6.4, 4.6.7, 4.6.8)
In problem settings, using abilities that comprise the power base, the student should be able to:

5. Compare whole numbers, fractions, and decimals to each other with and without models
    a. compare whole numbers (up to and including seven digits) with and without models
    b. compare two proper fractions with models
    c. compare proper fractions that have the same denominators without models
    d. compare two decimals (up to and including two decimal places) with and without models
6. Recognize and determine equivalent forms of a number with and without models
    a. recognize and determine equivalent forms of a proper fraction with and without models
    b. recognize and determine equivalent forms of a decimal (up to and including two decimal places) with and without models
7. Order whole numbers and decimals with and without models
    a. order a maximum of four whole numbers (up to and including seven digits) from least to greatest or vice versa
    b. order a maximum of four decimals (up to and including two decimal places)

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Model whole numbers, fractions, and decimals using cuisinaire rods, fraction bars, fraction circles, place value blocks, fraction stackers, or pattern blocks
2. Create diagram of models on graph paper

OPEN-ENDED QUESTION:

On Friday, your class will have a party after lunch. Each of the 30 students in your class has chosen one party activity. Here are the results:

> $\frac{1}{2}$ of the class chose outdoor relay races.
>
> $\frac{1}{3}$ of the class chose indoor games.
>
> The rest of the class chose to watch a movie.

- How many students chose to watch a movie?
- Show all of your work and explain your answer.

SCORING RUBRIC:

3 points	The student determines that 5 students chose to watch a movie, show how the answer was found, and gives a clear and logical explanation detailing how the problem was solved.
2 points	The student determines that 5 students chose to watch a movie and provides a vague or incomplete explanation of an appropriate process for solving the problem.
Or	The student, due to a minor error, gives an incorrect answer for the number of students who chose to watch a movie but gives a clear and logical explanation of how the problem was solved.
1 point	The student attempts to find the number of students who chose to watch a movie and may find at least one of the answers (5, 10, or 15) correctly, showing some understanding of the problem. However, the student's work shows major errors, incomplete procedures, or an incomplete explanation.
0 points	The response show insufficient understanding of the problem's mathematical concepts

MACRO (D): (4.8.1, 4.8.7)
Demonstrate an understanding of the meaning of the four basic arithmetic operations through modeling and discussion.

KNOWLEDGE: (4.8.1, 4.8.7)
The student should have a conceptual understanding of:

1. Meanings of basic arithmetic operations (+), (-) (X or *), (÷ or /)
2. Properties of operations
    a. zero (additive, multiplicative)
    b. one (multiplicative)
    c. grouping (associative)
    d. Order (commutative)
3. Inverse operations
    a. addition and subtraction
    b. multiplication and division
4. Relationship between addition and multiplication
5. Relationship between subtraction and division

The student should be able to:

6. Use models to represent and explain the meaning of each of the four basic arithmetic operations and their properties.
7. Use models to represent and explain inverse operations.
8. Use models to represent and explain the relationship between addition and multiplication.
9. Use models to represent and explain the relationship between subtraction and division.

PROBLEM-SOLVING SKILLS: (4.8.1, 4.8.7)
In problem settings, using abilities that comprise the power base, the student should be able to:

10. Choose the appropriate operation (s) to solve a problem.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build sets and arrays to demonstrate properties, inverses, and relationships of operations using centimeter cubes, color tiles, unifix cubes
2. Create diagrams of sets and arrays using graph paper
3. Use models to demonstrate division as partitioning and sharing
4. Model properties of operations
5. Model multiplication as repeated addition or same size grouping

OPEN-ENDED QUESTION:

You have 450 pennies. You wrap them in penny wrappers that fit 50 pennies in each wrapper.

- How many wrappers will you fill?
- How much money do you have?

(Show your work and clearly explain your answer.)

SCORING RUBRIC:

3 points	The student determines that 9 wrappers are needed, he has $4.50, and the explanation is clear demonstrating knowledge of decimals and division.
2 points	The student determines that 9 wrappers are needed, he has $4.50 and provides a vague or incomplete answer.
Or	The student, due to a minor error gives an incorrect part of the answer, but, gives a clear and logical explanation of how the problem was solved.
1 point	The student attempts to give the correct number of papers and total amount of money, showing some understanding, however, the work shows errors in procedures or explanations are incomplete.
0 points	The response shows insufficient understanding of the problem's mathematical concepts.

MACRO (E): (4.8.2, 4.8.3)
Use and explain paper and pencil procedures for performing whole number calculations.

KNOWLEDGE: (4.8.2, 4.8.3)
The student should have a conceptual understanding of:

1. Number facts
    a. addition (sums to 18)
    b. subtraction (inverse of addition)
    c. multiplication (up to and including 10 x 10)
    d. division (inverse of multiplication)
2. Place value and regrouping
3. Arithmetic computation
    a. addition (up to and including three 3-digit addends)
    b. subtraction (up to and including three digits)
    c. multiplication (up to and including two 2-digit factors)
    d. division (up to and including three digits divided by a one-digit divisor, with and without remainders)

PROBLEM-SOLVING SKILLS: (4.8.2, 4.8.3)
In problem settings, using abilities that comprise the power base, the student should be able to:

4. Use and explain procedures for performing whole number computations.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build number fact arrays using color tiles, centimeter cubes, etc.
2. Create array diagrams on graph paper
3. Model number fact families using counters
4. Build 1,000's numbers using place value blocks
5. Model regrouping using blocks
6. Create diagram to show regrouping on centimeter graph paper
7. Model multiplication and division using centimeter cubes and place value blocks

OPEN-ENDED QUESTION:

- Write a number in each line to complete this loop:

  $48 \div 6 = \_\_\_ \times 15 = \_\_\_ + 20 = \_\_\_ \div 7 = \_\_\_ \times 9 = \_\_\_ \div 5 = \_\_\_ + 12 = 48$

  (May be displayed as a loop coming full circle.)

- Make up your own loop with at least 2 multiplication and 2 division problems.

SCORING RUBRIC:

3 points	The student completes all the missing number on the loop, then creates a loop wit 2 multiplication and 2 division problems showing clear understanding of the arithmetic concepts.
2 points	The student successfully completes the loop, however, their own loop has less then four problems or a minor computation error.
Or	The student makes a minor mistake in the missing numbers on the loop, however, successfully completes their own 4-part loop.
1 point	The student attempts to complete the loop and create their own, however, makes errors in both procedures.
0 points	The response shows insufficient understanding of the problem's mathematical concepts.

MACRO (F): (4.8.5, 4.8.6)
Select and use appropriate whole number computational methods and check the reasonableness of the results.

KNOWLEDGE: (4.8.5, 4.8.6)
The student should have a conceptual understanding of:

1. Variety of mental computation strategies
    a. number facts
    b. mental math
    c. estimation
2. Calculator use
3. Paper and pencil procedures

The student should be able to:

4. Demonstrate proficiency with number facts using a variety of fact strategies.

PROBLEM-SOLVING SKILLS: (4.8.5, 4.8.6)
In problem settings, using abilities that comprise the power base, the student should be able to:

5. Select, use, and explain an appropriate computational method (mental math, paper and pencil, estimation, calculator) to solve a problem.
6. Determine the reasonableness of an answer.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:
Number Facts and Computation

1. Build arrays using unifix cubes, centimeter cubes, color tiles
2. Shade arrays on graph paper to model multiplication facts
3. Use number line to identify location of a number
4. Model rounding by identifying location on number line

OPEN-ENDED QUESTION:

Mrs. Brown's fourth grade class was collecting plastic bottles for a recycling project. They collected 53 bottles on Monday, 62 bottles on Wednesday, and 56 bottles on Friday. Their goal was to collect at least 150 bottles over the 3 days.

- Did they meet their goal? _____

  (Show your work and clearly explain your answer.)

SCORING RUBRIC:

3 points	The student verifies that the class did meet their goal demonstrating correct computation and clearly explains the steps in the process.
2 points	The student verifies that the class did meet their goal, however, offers a vague or incomplete explanation.
Or	The student, due to a minor computation error, does not verify that the class met their goal, however, gives a clear explanation of how the problem was solved.
1 point	The student shows part of the necessary computation and offers an incomplete explanation of their work.
0 points	The response shows insufficient understanding of the problem's mathematical concepts.

MACRO (G): (4.8.4)
Perform operations with commonly used fractions and decimals, using models.

KNOWLEDGE: (4.8.4)
The student should be able to:

1. Model fraction addition and fraction subtraction.
2. Model decimal addition and decimal subtraction.
3. Model decimal multiplication and decimal division involving one whole number and one decimal.

PROBLEM-SOLVING SKILLS: *(not applicable)*

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:
Model Fractions - Addition and Subtraction

1. Build fractions using pattern blocks, fraction bars or circles, paper folding, or Cuisinare rods.
2. Demonstrate addition and subtraction by exchanging pieces to create like fractions.

Model Decimal Computation

1. Build decimals on place value 100 tiles
2. Build decimals with Cuisinare rods
3. Demonstrate addition and subtraction by exchanging ten-sticks and ones-cubes on 100 tile.
4. Model multiplication and division using place value pieces on centimeter graph paper.

OPEN-ENDED QUESTION:
Ted does not believe that $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ or that $\frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

- Draw a picture for Ted showing why $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

  (You may want to use your colored shapes. Explain your answer.)

- Draw a picture for Ted showing why $\frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

  (You may want to use your colored shapes. Explain your answer.)

SCORING RUBRIC:

3 points — The student correctly demonstrates why $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

and $\frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

Explanations may be vague, but the pictures are sufficient to show understanding of both operations.

2 points — The student correctly demonstrates why EITHER $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Or $\frac{5}{6} - \frac{1}{6} = \frac{2}{3}$

Pictures are sufficient to show understanding.

1 point — The response fails to meet the requirements of a "2" but explanations are sufficient to show partial understanding of at least one of the computations.

0 points — The response shows limited to no understanding of the problem's mathematical concepts.

MACRO (H): (4.6.6)
Count and perform simple operations using money.

KNOWLEDGE: (4.6.6)
The student should have a conceptual understanding of:

1. Coins and paper money and their corresponding values.

The student should be able to:

2. Find the value of a collection of coins and paper money.

PROBLEM-SOLVING SKILLS: (4.6.6)
In problem settings, using abilities that comprise the power base, the students should be able to:

3. Solve problems involving money.
    a. determine total cost
    b. determine change
    c. determine different ways to represent money values

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Use play money to model groups of coins and bills.
2. Use coin stamps to create specified amounts of money or groups of coins.
3. Create diagrams of play money models.
4. Use newspaper ads to model shopping

OPEN-ENDED QUESTION:
Michelle volunteered to go to the store for her mother. She had to purchase 4 apples at $0.25 each, 2 peppers at $0.50 each, and 1 loaf of bread at $1.50.

- How much change would she get from a five dollar bill?
  (Show your work and clearly explain your answer.)

SCORING RUBRIC:

3 points	The student determines that $1.50 change would be given a and that the total cost is $3.50. The explanation is clear and logical.
2.points	The student correctly identified the total cost of $3.50 and identifies the amount of change, however, the explanation is vague or incomplete.
Or	The student attempts to tell the amount of change from the five dollar bill, providing a clear and concise explanation.
1 point	The student attempts to identify the cost and change, showing some understanding, however, the work shows errors or the explanations are incomplete.
0 points	The written response shows little understanding of the

problem's mathematical concepts.

CLUSTER II: MEASUREMENT

Informed By Content Standards: 4.9

Informed By Cross-Content Standards:4.1, 4.2, 4.3, 4.4, 4.5, 4.10, 4.16 Power Base

MACRO (A): (4.9.1, 4.9.2, 4.9.3, 4.9.5, 4.9.6)

Select and use appropriate non-standard and standard units of measure to describe, compare, and order various quantities.

KNOWLEDGE: (4.9.1, 4.9.2, 4.9.3, 4.9.5, 4.9.6)

The student should have a conceptual understanding of:

1. Measurable attributes
    a. length, distance, area, volume
    b. capacity, weight, time, temperature
2. Non-standard units of measure
3. Standard units of measure
    a. all customary units (excluding bushel and peck)
    b. all metric units (excluding deka- and hecto-)
4. Personal referents that approximate standard units of measure

The students should be able to:

5. Recognize the need for a standard unit of measure
6. Use rulers, scales, thermometers, and clocks to accurately measure length, weight, temperature, and time

PROBLEM-SOLVING SKILLS: (4.9.1, 4.9.2, 4.9.3, 4.9.4, 4.9.5, 4.9.6)

In problem settings, using abilities that comprise the power base, the student should be able to:

7. Select the appropriate unit of measure for length, weight, capacity, temperature, area, and volume
    a. customary
    b. metric
8. Select the appropriate unit for measuring time
9. Select the appropriate measurement tool for a given situation
10. Use personal referents to approximate measures
11. Solve problems involving measurement
    a. perimeter of a polygon
    b. area of a rectangle by grid or given formula
    c. area of other polygons by grid
    d. area of irregular regions by grid
    e. volume of a rectangular solid by counting cubes
    f. time
    g. temperature
    h. liquid capacity
    1. weight
12. Compare and order objects according to a measurable attribute.

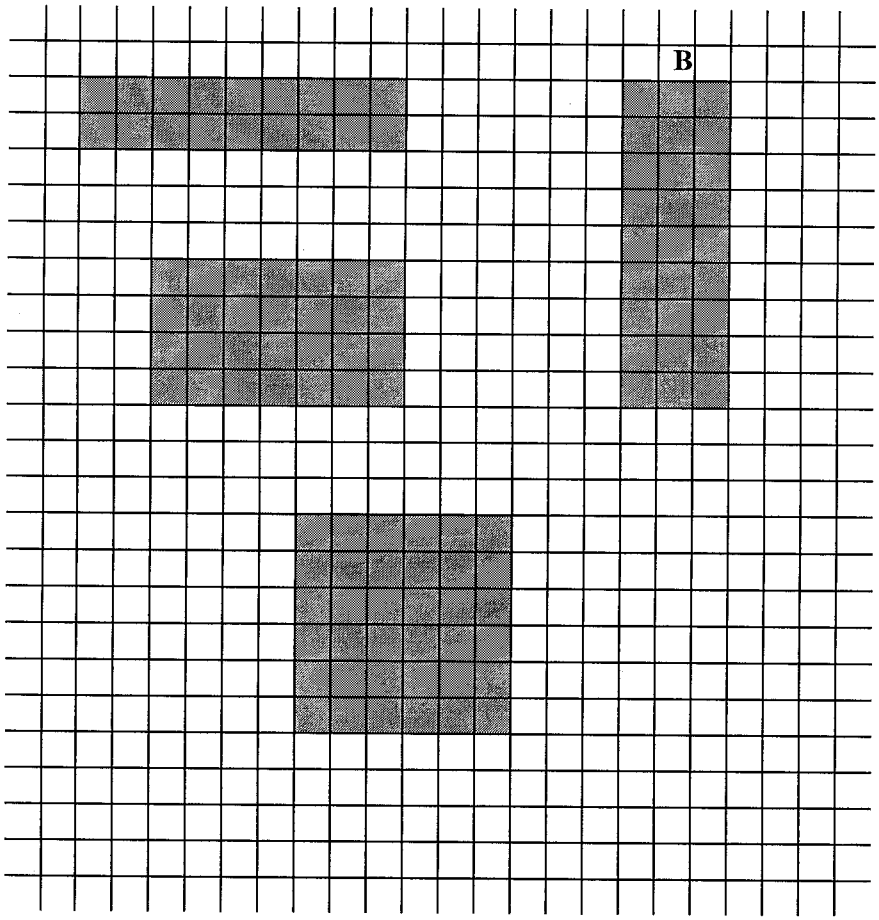
DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

Measurable Models

1. Build polygons using geoboards, color tiles, pattern blocks
2. Create diagram of polygons using cheesebox dot paper or centimeter graph paper
3. Model a square as having the characteristics of a rectangle.

OPEN-ENDED QUESTION:

Carefully examine each of the four rectangles shown below.



- Compute the perimeter of each rectangle
- Determine which rectangle has the greatest area

(Show your work and clearly explain your answer.)

SCORING RUBRIC:

3 points	The response shows complete understanding of the problem's essential mathematical concepts by correctly computing the perimeters and identifying that rectangle D, which is a square, has the greatest area. The area can be stated in terms of rectangle D having more squares than rectangles A, B, or C. The response does not have to state the area as 36 square units. The response contains a clear, effective explanation detailing how the problem was solved.
2 points	The response shows nearly complete understanding of the problem's essential mathematical concepts. The student will correctly compute the perimeter and determine the area of a majority of the rectangles shown. The response may contain minor mathematical errors in computation, but will demonstrate a clear understanding of the concepts of perimeter and area. The explanation detailing how the problem was solved may not be clear, and may cause the reader to make inferences.
1 point	The response shows limited understanding of the problem's essential mathematical concepts by incorrectly computing the perimeter and area of the majority of the rectangles shown. The explanation may be missing or may clearly demonstrate the student does not understand the concepts of perimeter and area.
0 point	The response show insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, will contain major errors. The explanation, if offered, will be unclear, incorrect, or irrelevant.

CLUSTER III: SPATIAL SENSE AND GEOMETRY

Informed By Content Standards: 4.7, 4.15.3

Informed By Cross-Content Standards:4.1, 4.2, 4.3, 4.4, 4.5, 4.10, 4.16 Power Base

MACRO (A): (4.7.3, 4.7.4)

Describe and use the properties and relationships of 2- and 3- dimensional shapes.

KNOWLEDGE: (4.7.3, 4.7.4)

The student should have a conceptual understanding of:

1. Basic geometric elements
    a. point, line, line segment, ray
    b. intersect, parallel, perpendicular
2. Standard notations $\overline{AB}$, $\overrightarrow{AB}$, $\overleftrightarrow{AB}$, $\angle ABC$
3. Basic 2-dimensional geometric terms, shapes, parts of shapes, and angles
    a. Polygon, triangle, guadrilateral, square, parallelogram, rectangle, trapezoid, rhombus, pentagon, hexagon, octagon, decagon
    b. side, vertex, diagonal
    c. circle, diameter, radius
    d. angle, right, acute, obtuse, straight, vertex
4. Basic 3-dimensional geometric terms, shapes, and parts of shapes
    a. sphere, cube, cone, pyramid, rectangular prism, cylinder
    b. edge, face, vertex, base
5. Two-dimensional properties
    a. angle classification
    b. number of sides
    c. curves or line segments
6. Three-dimensional properties
    a. number of faces, edges, vertices

PROBLEM-SOLVING SKILLS: (4.7.3, 4.7.4)

In problem settings, using abilities that comprise the power base, the student should be able to:

7. Use properties, definitions, and relationships to identify, classify, and describe 2-dimensional geometric shapes
    a. number of sides
    b. classification of angles
    c. curves, line segments
8. Use properties, definitions, and relationships to identify, classify, and describe 3-dimensional geometric shapes
    a. number of faces, edges, vertices
9. Use manipulatives to demonstrate properties of 2-dimensional and 3-dimensional shapes
    a. liquid capacity
    b. weight

10. Compare and order objects according to a measurable attribute

DEVELOPMENTAL ACTIVITIES AND MANIUPLATIVES:

1. Build geometric elements and 2-dimensional shapes on geoboard
2. Model plane figures by using tangrams, pentominoes, or color tiles
3. Create diagrams using graph paper shading
4. Identify silhouettes of 3-dimensional wooden shapes to investigate edge, face and base of shapes
5. Use unifix blocks or centimeter cubes to build 3-dimensional shapes
6. Use 3-dimensional paper patterns, or "nets", to build shapes

OPEN-ENDED QUESTION:

Colin cut 6 figures out of construction paper and put them in a box.

(Display: circle, pentagon, hexagon, triangle, square, quadrilateral with name printed below)

Without looking, he randomly pulled one from the box. Use the clues below to find out what Colin chose.

CLUES

The figure is made of line segments
The figure has more than 3 sides but fewer than 6 sides
The figure has at least one line of symmetry
The figure is not a square

- What figure did Colin choose?
- Explain how you got your answer.

SCORING RUBRIC:

3 points	The student correctly identifies the pentagon. The explanation conveys clear and logical knowledge of the other shapes and understanding of the knowledge of 2-dimensional shapes.
2 points	The student correctly identifies the pentagon, however, does not give a clear explanation of the other shapes and convey an understanding of concepts.
Or	The student does not identify the correct shape, however, gives a clear explanation of the other shapes and understanding of concepts.
1 point	The student attempts to identify the shape, however, is incorrect and shows little understanding in the explanation.
0 points	The response shows insufficient understanding to be able to identify any of the shapes.

MACRO (B): (4.7.8)
Use coordinates and paths in maps, tables, and grids

KNOWLEDGE: (4.7.8)
The student should have a conceptual understanding of:

1. Methods of locating points in the plane.
    a. point, coordinates, ordered pair
    b. horizontal, vertical, latitude, longitude
    c. scale, table, grid

The student should be able to:

2. Identify the coordinates of a point on a grid or map.
3. Plot a point given its coordinates

PROBLEM-SOLVING SKILLS: (4.7.8)
In problem settings, using abilities that comprise the power base, the student should be able to:

4. Construct a simple figure by connecting points given a series of ordered pairs.
5. Use data from a table or grid to solve problems.
6. Interpret information from a map, grid, or table.
7. Construct a grid or table using data.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Model coordinate grid on geoboard
2. Model points on globe
3. Chart locations on map
4. Play game of "Battleship"
5. Identifying coordinate pairs on grid
6. Create diagram of coordinate grid on graph or dot paper
7. Label axis and locate items on grid

OPEN-ENDED QUESTION:

The grid below is part of a map. Each square on the grid is one square block.

Display a 10 x 10 grid labeled – North, South, East, and West. Also shown are the following: the Zoo in grid block (3, 6); the Bus Stop in grid block (6, 2); the Sport Shop in grid (10, 10)

- Put each of the following in the correct grid square:
  - The Library is 5 blocks north and 3 blocks east of the Zoo
  - The Theatre is 6 blocks north and 2 blocks west of the Bus Stop

- Give directions from the Bus Stop to the Sports Shop.
- Give directions from the Sport Shop to the Zoo, stopping at the Theatre on the way.

SCORING RUBRIC:

3 points	The student correctly plots the Library at square (5, 9) and the Theatre at square (8, 5). Also, given are written directions to travel from the Bus Stop to the Sports Shop and from the Sport Shop to the Zoo, via the Theatre using the correct directional words and counting of grid squares.
2 points	The student correctly identifies the location of the Library and Theatre, however, due to a minor counting error, fails to give accurate directions from the Bus Stop to the Sport Shop and from the Sport Shop to the Zoo
Or	The student, due to a minor counting error, fails to locate either the Library or Theatre, however, is able to give accurate directions from the Bus Stop to the Sport Shop.
1 point	The student successfully locates one location or gives accurate directions for one location.
0 points	The student demonstrates no accuracy in locating or giving any directions, demonstrating, insufficient understanding of the concepts.

MACRO (C): (4.7.1, 4.7.2, 4.7.7, 4.7.10)
Use transformations to determine spatial relationships among 2-dimensional figures.

KNOWLEDGE: (4.7.1, 4.7.2, 4.7.7, 4.7.10)
The student should have a conceptual understanding of:

1. Geometric transformations and related properties.
    a. flip, turn, slide, stretch, shrink
    b. congruence, similarity, symmetry
    c. tessellation

The student should be able to:

2. Identify transformations in nature, art, and other areas

PROBLEM-SOLVING SKILLS: (4.7.1, 4.7.2, 4.7.7, 4.7.10)
In problem settings, using abilities that comprise the power base, the student should be able to:

3. Determine lines of symmetry.
4. Identify congruent figures.
5. Use transformations to draw the image of figure.
6. Identify the transformation used given a figure and its image.
7. Classify a variety of 2-dimensional shapes by congruence and/or similarity.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build coordinate grid on geoboard.
2. Create pattern tessellations using pattern blocks.
3. Model and create geometric transformations using tangrams, pattern blocks, pentominoes, and color tiles.
4. Model congruent figures and transformations using pattern blocks.
5. Display geometric transformations on graph or grid paper.

OPEN-ENDED QUESTION:

Use the grid provided for the following problem:

- Plot and label the points listed below.
  B (11, 7) C (8, 12) D (2, 12)

(Display grid, with even numbers only, shown on alternate lines of the x and y axis and point A shown and labeled at coordinates (4, 6).

- Connect points A,B,C and D to draw figure ABCD.
- Use the colored shape that matches the figure ABCD above. Flip your shape over the Line AB. Trace the shape.
- Is the flipped shape congruent with the original shape?

SCORING RUBRIC:

3 points	The student accurately plots the points, sketches the parallelogram, performs the flip, and states that the shapes are congruent.
2 points	The student accurately plots the points, or may attempt to plot the points, but may have a minor error. The student attempts to sketch the image of the flipped parallelogram and may or may not state that the shapes are congruent.
Or	The student accurately plots the points, states that the shapes are congruent but does not perform the flip.
1 point	The student accurately plots the points, but fails to recognize that the shapes are congruent.
Or	The student misplots more than 1 point, but correctly states the shapes are congruent.
0 points	The response shows insufficient understanding of the problem's mathematical concepts.

MACRO (D): (4.7.5, 4.7.6, 4.7.9, 4.15.3)
Demonstrate the ways in which geometric shapes and objects can be measured, combined, subdivided, and changed.

KNOWLEDGE: (4.7.5, 4.7.6, 4.7.9, 4.15.3)
The student should have a conceptual understanding of:

1. Measures of shapes
    a. area, perimeter, volume

PROBLEM-SOLVING SKILLS: (4.7.5, 4.7.6, 4.7.9, 4.15.3)
In problem settings, using abilities that comprise the power base, the student should be able to:

2. Predict the resulting shape when combining, subdividing, and changing figures.
3. Estimate the area of a region by identifying familiar subdivisions.
4. Determine the perimeter of a polygon made up of unit squares.
5. Find possible perimeters for a given area made up of unit squares and vice versa.
6. Determine the number of cubes in a solid using models.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build plane figures on geoboard or using color tiles
2. Create diagram of 2-dimensional figures on graph or grid paper
3. Build 3-dimensional shapes using unifix cubes or centimeter cubes
4. Build plane figures using many variations of pattern blocks
5. Build plane figures using many variations of tangram pieces

OPEN-ENDED QUESTION:

The area of 1 trapezoid is 3 square units. (Display 1 trapezoid.)

- What is the area of each of the following shapes?
  _____ square units
  _____ square units

  (Display a figure made by tracing 3 hexagons with sides touching and a large trapezoid made by 4 small trapezoids with sides touching.)

- Explain how you got your answers.

SCORING RUBRIC:

3 points	The student correctly identifies the area of the 3 hexagon shapes as equaling 18 square units and the area of the large trapezoid as equaling 12 square units and successfully explains the procedure by using the area of the small trapezoid and multiplying or demonstrating repeat addition.
2 points	The student correctly identifies the areas of the 2 large shapes, however, fails to explain the procedure.
Or	The student fails to correctly identify the area of 1 of the shapes due to minor computation errors, however, fully explains understanding of the procedure.
1 point	The student fails to identify the area of either of the shapes and does not clearly explain the procedure.
0 point	The response demonstrates insufficient understanding of the procedure or content.

CLUSTER IV: DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

Informed By Content Standards: 4.12, 4.14
Informed By Cross-Content Standards:4.1, 4.2, 4.3, 4.4, 4.5, 4.10, 4.16 Power Base

MACRO (A): (4.12.2, 4.12.6, 4.12.7, 4.12.8)
Determine the probability of a simple event and predict outcomes.

KNOWLEDGE: (4.12.2, 4.12.7, 4.12.8)
The student should have a conceptual understanding of:

1. Probability and related vocabulary
    a. event, chance, outcome, fairness, randomness
    b. possible, impossible, certain, probable, equally likely, fair, fairness, chance

The student should be able to:

2. Determine the probability of simple events.

PROBLEM-SOLVING SKILLS: (4.12.2, 4.12.6, 4.12.7, 4.12.8)
In problem settings, using abilities that comprise the power base, the student should be able to:

3. Solve problems that involve predicting the outcome of a change event.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Play games using spinners displaying various size sections
2. Choose blocks, beads, color tiles, etc., from box or bag
3. Record outcome using a tally or graph
4. Compare various groups of block, beads, color tiles, etc., to identify "best chance" of choosing a certain color
5. Display spinner and item probability as fraction

OPEN-ENDED QUESTION:

It is Cheryl's turn to spin a spinner to choose an activity for her physical education class. Cheryl will spin one of the spinners below to decide what the class will do.

(Display 2 circle spinners. Spinner 1 is cut in fourths with Soccer, Swimming, Bowling, and Running each written in 1 section. Spinner 2 is cut in thirds with Bowling, Swimming, and Soccer each written in 1 section.)

Cheryl's favorite activity is swimming.

- Find the probability of landing on swimming using Spinner 1.
- Find the probability of landing on swimming using Spinner 2.
- Decide which spinner Cheryl should choose if she wants to go swimming.
- Explain why Cheryl should choose this spinner.

SCORING RUBRIC:

3 points	The student appropriately responds to all four parts of the question. Identifying Spinner 1 as 1 /4 and Spinner 2 as 1/3 and explains their use of probability.
2 points	The student correctly finds the probability of landing on swimming for both Spinner 1 and Spinner 2.
Or	The student correctly finds the probability of landing on swimming for either Spinner 1 or Spinner 2, and gives a reasonable explanation for which spinner Cheryl should choose.
1 point	The student correctly finds the probability of landing on swimming for either Spinner 1 or Spinner 2.
Or	The student gives reasonable explanation for which Cheryl should choose.
0 point	The response shows limited to no understanding of the problem's mathematical concepts.

Note: A reasonable explanation for bullet four might include a bigger area, fewer choices, or comparison of fractions.

MACRO (B): (4.12.1, 4.12.3, 4.12.4, 4.12.5, 4.14.4)
Collect, organize, analyze, and interpret data.

KNOWLEDGE: (4.12.1, 4.12.3, 4.12.4, 4.12.5, 4.14.4)
The student should have a conceptual understanding of:

1. Measures of central tendency and descriptions of sets of data.
    a. mean, median, mode, average
    b. frequency, data, survey, range, tally
2. Displays of data.
    a. frequency tables
    b. circle graphs, pictographs, bargraphs, broken line graphs

PROBLEM-SOLVING SKILLS: (4.12.1, 4.12,3, 4.12.4, 4.12.5, 4.14.4)
In problem settings, using abilities that comprise the power base, the student should be able to:

3. Represent data using an appropriate graph.
4. Analyze data using mean, median, mode, range, frequency.
5. Make inferences based on organized data and displays.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Compare and arrange real objects in increasing or decreasing order. (i.e., people, various size sticks, test scores)
2. Use real objects to model central tendency of sets.
3. Model bar and line graph on geoboard or by using color tiles or unifix cubes.
4. Model circle graph using fraction circles.
5. Diagram graphs on graph or dot paper.

OPEN-ENDED QUESTION:

After lunch, fourth-grade students at Washington Elementary School choose an activity. Last week, their choices were basketball, relay races, soccer, or reading. The table below shows the choices that each class made.

Number of Students Choosing Each Activity

Class	Activity: Basketball	Relay Races	Soccer	Reading
Mr. Green	10	7	8	9
Mr. Batista	7	10	9	8
Mr. Kelly	11	6	4	9
Ms. Flemming	8	9	3	10

- Use the information from the table above to create a bar graph of the choices made in Ms. Flemming's class. Be sure to label all parts of the graph and give the graph a title.

SCORING RUBRIC:

3 points	The student creates an accurate bar graph. The graph includes all appropriate labels and title, and show the votes as follows: 8 for basketball, 9 for relay races, 3 for soccer, 10 for reading.
2 points	The student creates a bar graph, but chooses a different class or omits axis labels and/or a title.
Or	The student creates a bar graph with appropriate labels and title, but the student makes an error in the size of one of the bars on the graph.
1 point	The student attempts to create a bar graph and shows some understanding of the problem, but the graph shows major errors.
Or	The student creates a correct graph that is not a bar graph.
0 points	The response shows limited to no understanding of the problem's mathematical concepts.

MACRO (C): (4.14.5)
Follow, devise, and describe algorithms.

KNOWLEDGE: (4.14.5)
The student should be able to:

1. Follow an algorithm to accomplish a given task.

PROBLEM-SOLVING SKILLS: (4.15.5)
In problem settings, using abilities that comprise the power base, the student should be able to:

2. Develop an algorithm.
3. Describe a set of instructions to accomplish a given task.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build large numbers using place value blocks and use to model addition and subtraction with regrouping.
2. Model large number multiplication and division using place value blocks.
3. Model numbers to thousands using graph paper shadings on 100 flats

OPEN-ENDED QUESTIONS:

In July, the zoo had 4,653 visitors. The next month, the zoo had twice as many visitors. In September, the number of visitors dropped to 979. How many people visited over the three-month period?

Explain your answers.

SCORING RUBRIC:

3 points	The student correctly identifies the number of visitors as being 14,938 and explains the doubling concept and subsequent computing correctly.
2 points	The student correctly identifies the number of visitors, however, gives a weak explanation of the computation.
Or	The student, due to a minor computation error, does not give the correct total, however, gives a thorough explanation.
1 point	The student gives an incorrect answer due to minor mistakes with an incomplete explanation.
0 points	The response show insufficient understanding of the problem and concepts.

MACRO (D): (4.14.1, 4.14.2)
Devise and use strategies for puzzles, games, and counting problems, as well as problems involving combinations and permutations.

KNOWLEDGE: (4.14.5)
The student should have a conceptual understanding of:

1. Network
2. Tree diagram

The student should be able to:

3. Make an organized list for simples counting problems.
4. Make an organized list of all possible combinations or permutations for a simple situation.

PROBLEM-SOLVING SKILLS: (4.14.1, 4.14.2)
In problem settings, using abilities that comprise the power base, the student should be able to:

5. Draw and interpret network and tree diagrams.
6. Compare and evaluate different strategies for playing a game.
7. Devise and apply strategies for solving puzzles and games.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Play probability coin and dice toss games
2. Record variable outcomes using tree diagrams
3. Use real objects to model combinations and variable outcomes (i.e., snack foods, menu items, articles of clothing, coins and bills, color beads, etc.)
4. Solve logic puzzles by recording clues on table or scatter plot grid
5. Use visual clues to recognize relationships (i.e., deduce volume of objects using balance scale equivalencies)

OPEN-ENDED QUESTIONS:

A juice machine charges $0.65 for a can of juice and accepts only nickels, dimes, or quarters. The machine requires exact change.

You have 4 nickels, 4 dimes, and 4 quarters. Make a table and list the different ways you can use your coins to make up exactly $0.65.

Show your work and clearly explain your answer.

SCORING RUBRIC:

3 points

The response shows complete understanding of the problem's essential mathematical concepts. The table will be complete and correct, listing all possible combinations of nickels, dimes, and quarters that can be used to purchase a can of juice.

One possible representation is:

Nickels	Dimes	Quarters	Total
0	4	1	$0.65
1	1	2	$0.65
2	3	1	$0.65
3	0	2	$0.65
4	2	1	$0.65

A clearly labeled table can serve as an explanation.

2 points

The response shows nearly complete understanding of the problem's essential mathematical concepts. The majority of the table will be completed accurately, but one or two possible combinations may be missing.

Or

The table will be complete but will have mathematical errors and any explanation offered will be unclear.

1 point

The response shows limited understanding of the problem's essential mathematical concepts. The table will contain less than three but at least one correct combination and/or may have major errors.

0 point

The response shows insufficient understanding of the problem's essential mathematical concepts. The table will be both incomplete and inaccurate, containing major errors. There may be no explanation of how and why decisions were made, or the explanation offered will be illogical, unclear, unclear, or not relevant.

CLUSTER V: PATTERNS AND ALGEBRA

Informed By Content Standards: 4.11, 4.13, 4.14.3, 4.15
Informed By Cross-Content Standards:4.1, 4.2, 4.3, 4.4, 4.5, 4.10, 4.16 Power Base

MACRO (A): (4.11.1, 4.11.3, 4.11.5, 4.14.3, 4.15.1)
Use a variety of materials to extend, create, and describe patterns, sequences, and relationships that are in mathematics and other disciplines.

KNOWLEDGE: (4.11.1, 4.11.3, 4.11.5, 4.14.3, 4.15.1)
The student should be able to:

1. Recognize patterns, sequences, and relationships including those found in nature and art.
2. Recognize that most patterns and sequences continue indefinitely.
3. Use concrete, pictorial, and symbolic representations to model patterns and relationships.

PROBLEM-SOLVING SKILLS: (4.11.1, 4.11.3, 4.11.5, 4.14.3, 4.15.1)
In problem settings, using abilities that comprise the power base, the student should be able to:

4. Reproduce and extend numerical and non-numerical patterns.
5. Create numerical and non-numerical patterns.
6. Describe numerical and non-numerical patterns.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Build linear and border patterns using color tiles, pattern blocks, etc.
2. Identify and shade in skip-counting patterns on the 100 Chart
3. Create number patterns on the calculator by using the constant key
4. Identify and extend changing patterns using attribute shapes
5. Model growing patterns using pattern blocks or Cuisinare rods (i.e., staircases, walls, rectangles, squares)

OPEN-ENDED QUESTION:

Sally is making a necklace by stringing beads in a certain pattern. A section of the unfinished necklace is inside the box.

Based on the pattern shown, draw or describe in detail the section of the necklace that is inside the box.

Show your work and clearly explain your answer.

(Display a box with a string of dark and light beads going into the top of the box on one side and coming out of the top of the box on the other. The color beads which can be seen going into the box are as follows: 1 light, 1 dark, 1 light, 2 dark, 1 light, 3 dark, 1 light. The color beads coming out of the box are as follows: 2 dark, 1 light, 7 dark, 1 light, 8 dark)

SCORING RUBRIC:

3 points	The response accurately states that the pattern of beads on the portion of the necklace inside the box with a clear, detailed explanation of how this determination was made. The box should contain 4 colored beads, 1 white bead, 5 colored beads, 1 white bead, and 4 colored beads, in that order. White and colored beads alternate. Beginning with 1 of each bead, the number of colored beads increases by 1 for each repetition. I.e., 1 white, 1 colored, 1 white, 2 colored, 1 white, 3 colored, 1 white, 4 colored, 1 white, 5 colored, and so on. By examining the pattern it can be determined that 4 colored beads, 1 white bead, 5 colored beads, 1 white bead, and 4 of the next colored beads are in the box.
2 points	The response shows nearly complete understanding of the problem's essential mathematical concepts by correctly listing the pattern of beads inside the box. Any errors are minor. The explanation may be somewhat unclear, causing the reader to make inferences, but will be essentially accurate.
1 point	The response shows limited understanding of the problem by recognizing that a pattern exists, but incorrectly identifying the pattern. There will be an incomplete explanation that will cause the reader to make inferences.
0 points	The response show insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, will contain major errors. An explanation, if offered, will be unclear, illogical, or not relevant.

MACRO (B): (4.11.2, 4.11.6, 4.13.1, 4.13.2, 4.13.3, 4.13.4)
Use algebraic concepts and processes to form, describe, and verify generalizations based on observations of patterns and relationships.

KNOWLEDGE: (4.11.2, 4.11.6, 4.13.1, 4.13.2, 4.13.3, 4.13.4)
The student should have a conceptual understanding of:

1. Basic algebraic elements
    a. variable, expression, open sentence, rule

The student should be able to:

2. Construct an open sentence when given a rule.
3. Construct a graph from a table.

PROBLEM-SOLVING SKILLS: (4.11.1, 4.11.3, 4.11.5, 4.14.3, 4.15.1)
In problem settings, using abilities that comprise the power base, the student should be able to:

4. Create a rule based on input and output data.
5. Solve a simple open sentence involving one operation.
6. Translate among tables, rules, open sentences, and graphs.
7. Form, describe, and verify generalizations based on observations of patterns and relationships.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Use real objects to build patterns (i.e., color tiles, pattern blocks)
2. Use unifix cubes or centimeter blocks to model in-put/out-put patterns
3. Model letter code patterns using real objects
4. Diagram growth patterns
5. Name the rule for growth patterns

OPEN-ENDED QUESTION:

Mrs. Thompson's class recently completed a lesson on number patterns. Each student had to write a rule to describe a pattern of numbers and list some numbers in the pattern. Two examples are shown below.

Rule: Start with 2. Multiply each number by 2 to get the next number in the pattern.

Pattern: 2, 4, 8, 16, 32, ...

Rule: Start with 1. Add 1 to the first number, add 2 to the second number, add 3 to the third number, and so on.

Pattern: 1, 2, 4, 7, 11, ...

- Tony's pattern is show below. Write a rule to describe his pattern. 1, 4, 5, 8, 9, ...
- Write your own rule for a number pattern.

- Also, write the first five numbers in your pattern.

SCORING RUBRIC:

3 points	The student gives a valid rule to describe Tony's pattern. The student also creates a rule to describe his or her new number pattern and accurately lists the first five numbers in that pattern.
2 points	The student gives a valid rule for Tony's pattern and either describes a new number pattern without providing the first 5 numbers or lists 5 numbers in an identifiable pattern, but the pattern he or she made up does not fit the description.
Or	The student creates a rule to describe his or her new number pattern and accurately lists the first five numbers in that pattern.
1 point	The student gives a valid rule to describe Tony's pattern.
Or	The student either describes a new number pattern without providing the first 5 numbers or lists 5 numbers in an identifiable pattern, but the pattern he or she made up does not fit the description.
0 points	The response show insufficient understanding of the problem's mathematical concepts.
Or	The student lists five numbers without providing an indication of a rule.

MACRO (C): (4.11.4, 4.15.2)
Recognize and describe change in quantities.

KNOWLEDGE: (4.11.4, 4.15.2)
The student should be able to:

1. Recognize that certain quantities (such as temperature, bank balances, height) change over time.

PROBLEM-SOLVING SKILLS: (4.11.4, 4.15.2)
In problem settings, using abilities that comprise the power base, the student should be able to:

2. Describe how certain quantities change over time.
3. Explain how a change in one quantity can produce a corresponding change in another.

DEVELOPMENTAL ACTIVITIES AND MANIPULATIVES:

1. Monitor the daily temperature and record on a line graph
2. Monitor and measure plant growth, over time, and record data on a graph
3. Build growth patterns using centimeter cubes or real objects
4. Read and act out story problems involving fund growth using play money
5. Model growth patterns, such as doubling, using real objects

OPEN-ENDED QUESTION:

You have to choose between two different job offers for the same four-day period. The first job pays $5.00 each day for the 4 days. The second job pays $1.00 for the first day, and each day after the first, you will be paid twice the amount you received the previous day.

- Which job will pay you the greatest amount?
- Would your answer change if you were working for more than 4 days?

Show your work and explain your answer.

SCORING RUBRIC:

3 points	The response shows complete understanding of the problem's essential mathematical concepts. The student's response will clearly detail the total amount the student would earn in both jobs, and state that Job 1 will pay more money for the 4-day period, or give some other reasonable method to determine which job pays more.
2 points	The response shows nearly complete understanding of the problem's essential mathematical concepts by correctly computing one of the two situations presented, and providing an accurate and detailed explanation of which job would be the better choice under the circumstances.
1 point	The response show limited understanding of the problem's essential mathematical concepts. Computations of the total in both situations will be incorrect, but the conclusion will be accurate.
0 points	The response show insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors or are incomplete. No conclusion or explanation will be offered or the explanation may be unclear, illogical, or irrelevant.

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)

MATHEMATICS MACRO SKILL DEVELOPMENT

Grades K – 4

Joyce Scholz
Pennsauken Public Schools

INTRODUCTION

This booklet is designed to assist Kindergarten through fourth grade teachers in their content development of mathematics skills on the fourth grade ESPA. It includes a content domain outline based on the 5 Content Clusters and the Macro Skills from the New Jersey Mathematics Standards. It may be utilized to track skill development in a particular grade level, to assess student prior knowledge, and to view the scope of mathematics skills through the elementary grades.

The goal is to demonstrate the building of math skills through the elementary grade levels. Preparation for ESPA testing begins in Kindergarten and grows each year until the students reach fourth grade.

Included in the domain outline for each content cluster are student outcomes, activities, and manipulatives, which may be utilized in lesson planning and skill development. This may serve as a useful tool for multi-grade level preparation of students for ESPA.

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NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (A): Demonstrate meaning for whole numbers, negative integers, commonly used fractions, and decimals using physical materials, technology, and real-life experiences.					
KNOWLEDGE: The student should have the conceptual understanding of:					
1. Whole numbers	Count, model, write & identify number 1-20; use one-to-one correspondence to compare two groups; show groups that are more less & equal; count objects in a group & compare to another group; count, represent, identify & compare the number 0; match number with number word (0-10)	Count, build, read, write to 100; use place value ones & tens pieces; compare using >, <, =; estimate groups less than 100; give number which is 1 more & I less; match number word (0-20)	Count, build, read, write to 1,000; use place value ones/ tens /hundreds building pieces; write in place value chart; using standard notation with equation; compare using >, <, = symbols	Count, build, read, write to 10,000; use place value building pieces; compare using >, <, =; round to nearest 10,100, 1000; write standard form using expanded notation as equation	Use a "benchmark" or known amount to give meaning to an unknown amount; count, read, build, write numbers to one million; write standard form & expanded notation equation; read number in words and write in standard form; compare using >, <, = symbols

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
2. Negative integers				Use in °F and °C on thermometer	Locate negative numbers on number line; order & compare; use in °C & °F
3. Commonly used fractions (halves, thirds, fourths, sixths, eighths, tenths)	Identify halves of a whole item; describe both parts as being the same/equal; use objects and pictures	Identify 1/2,1/3,1/4, of a whole; identify 1/2,1/3,1/4, of a group of items; Use real objects (i.e. crackers, counters, etc.)	Identify fractional parts to twelfths of a whole & of a group; use 2-color chips, pie slices, counters, & circle side of geoboard	Model parts of a whole & parts of a group using pictures, pattern blocks & fraction bars	Model, read & write, fractions of a whole; model equivalent decimal fractions using fraction bars, centimeter cubes & Cuisinaire rods
4. Decimals (tenths, hundreds, values greater than one)			Relate decimals to cents in a dollar; use dollar sign & decimal point	Relate fractions to decimals; write tenths/hundredths using 100 grid; read/write decimals; compare like fractions	Model decimal fractions using place value pieces & grid diagram; model mixed numbers and fractions using centermeter graph paper

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Number line with integers, commonly used fractions and decimals		Use number line or 100 chart; find number between 68-70; find +10, -10, +1, -1 (calculator)	Use number line or 100 chart; find number before, after & between; round to nearest 10	Compare numbers to 1,000; model using number line & place value blocks & chart; compare/order decimals use fraction bars & place value chart	Use a number line & place value to compare & order numbers to millions; model equivalent fractions with fraction bars and number line; read & write fractions & decimal parts; use number line for rounding fractions & decimals
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
6. Use whole numbers, negative integers, commonly used fractions and decimals a. whole numbers (up to and including seven digits)	Count objects in a group to determine quantity; make groups using connecting cubes, bears, buttons, etc.; correctly form the numbers using sandbox writings; use calendar to identify one & two-digit numbers; identify number shown on tens counting frame	Build with connecting cubes; model with place value pieces	Build with connecting cubes & place value pieces; match number to expanded notation with words & numbers	Building with connecting cubes; place value pieces; match number to expanded notation equation	Write numbers in place values chart; change the value of a number by a given amount, then compare digits to find the value of the change; order groups of numbers to millions; write standard form from number words

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. negative integers using number line and/or thermometer				Use thermometer to identify below zero temperatures	Use thermometer to identify below zero temperatures; use number line to explain ordering of negative numbers
c. commonly used fractions	Model using shape; folding activity; color ½ of shape in diagram	Model using pattern blocks, geoboards; fraction pieces; counters, real objects (i.e. crackers)	Model fraction pieces to twelfths using fraction bars, pie pieces, pattern blocks, real objects; compare size of pieces	Model, write & compare fractions; use pattern blocks, fraction bars, real objects (i.e. pizza)	Model, write & compare fractions; use measuring cups markings, music notes, cuts & parts of real objects; Cuisinaire rods and pattern blocks to model
d. decimals (up to and including hundredths place)			Model the value of $1.00 using pennies & dimes	Model, write, compare decimals; use 100 grid, fraction bars & decimal number line	Model, compare, write decimals; model using cm. graph paper diagrams & place value chart to record

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (B): Show understanding of place value concepts and numeration using counting, groups, and patter identification					
KNOWLEDGE: The student should have the conceptual understanding of:					
1. Counting/place value relationship	Model numbers 0-20 using connecting cubes; count groups of ten items and place them on tens counting frame; estimate numbers by comparing to group of 10	Model double digit numbers using place value blocks; on counting mat	Model 3-digit number to 1,000; use place value blocks on mats; find value of a digit by using place value position	Write numbers in standard, expanded work form; find value of a digit by using place value positions and place value blocks	Model numbers to millions using place value chart; write using words, numerals & expanded notation
2. Grouping and place value	Count out items & group in tens; model 11-30 as groups of ten with extras using ten counting frame	Count by tens, build tens/ones numbers using manipulatives (i.e. craft sticks, unifix cubes, etc.)	Write & rename 3 digits numbers as groups of ones & tens using place value blocks	Write regrouping of tens numbers as expanded notation equation; model using base ten blocks	Model multiplication using base ten blocks; use blocks and diagrams

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Expanded notation and numeration		Build tens/ones to demo: 6 tens 6 ones = 66 60 + 6 = 66	Read & write to 3-digit numbers using place value chart; use connecting cubes	Model place value relationships in numbers to 10,000; use connecting cubes & place value pieces on place value mat	Find the value of the expanded notation equation
4. Number patterns and place value a. multiplication by 10, 100, 1000... (powers of ten)			Model multiplication by ten using counters & place value sticks	Model skip counting on number line & 100 chart; use basic facts & place value patterns to multiply by 10, 100, 1000	Model multiplication by ten on fact table; use facts & mental math to 2 & 3 digit numbers by 10, 100 & 1000
b. ten-to-one relationship of adjacent place values		Model with place value pieces 10 ones = 1 ten 10 tens = 100; use Arrow Math on 100 chart; +1→, -1←, +10↓, -10↑; compare digit values	Model to 1,000 numbers using place value pieces; find value of digit by building on workmat & chart; identify 1, 10, 100 more or less than a number; use Arrow Math	Model to 10, 000 numbers using place value pieces; find value of digit using place value charts,; identify up to 1,000 more or less than a number	Multiply the digit by its place value position to find its value
c. periods					Use "period" to define each group of 3 digits (i.e., ones, thousands); use place value position

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Rounding and place value			Round to nearest 100 using number line; use to estimate solutions in word problems	Round to nearest 10,100, 1,000 using number line; round to nearest dollar to estimate total; develop rule for rounding	Round whole numbers to a given place value using number line & mental math; round numbers to estimate answers for all operations ; round dollar amounts to estimate sums/differences
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the lower base, the student should be able to:					

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
6. Use place value concepts and numeration a. use skills in counting on and counting groups	Move ahead on game board; build a ten stick with connecting cubes then put on I more, illustrate; count on from 5¢ & 10¢, find total	Use cubes to build 2-digit numbers; extent number patterns; count by 10's on 100 chart; skip count by 2, 5; count 1¢, 5¢, & 10¢	Use place value pieces to build 3-digit numbers; skip count by 2,3,4,5,10 using manipulatives; count on from the price to find the change	Identify & extend counting on patters; use skip counting, growing patterns, decreasing pattern	
b. use models to demonstrate regrouping		Write 2 digit numbers in tens/ones chart; align for add/subtraction	Model a number using different combinations of ones, tens, hundreds	Rename numbers using expanded notation format; model with place value blocks	Model multiplication using base ten blocks
c. identify the value of a digit in a given number		Model groups of tens/ones; find sums & differences by counting up & back on 100 chart (i.e., 64 + 3)	Identify place value as multiples of ten; use place value picture to explain renaming of up to 100 numbers	Identify the value of a digit by building with place value blocks on a place value mat; identify place value relationships to 10,000 numbers	Identify the value of a digit by writing it in a place value chart
d. write a number in expanded form or standard form		Build to show value of digits; identify value of 8 in 84/48; Use >, <, = to compare (i.e., 6 tens 4 ones 065)	Write numbers to 1,000 as an expanded form equation; with place value labels or in standard form	Write numbers to 10,000 as an expanded form or in standard form; write numeral given in number work form	Write numbers to millions in standard or expanded form

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
e. round a whole number to the nearest ten, hundred, or thousand			Use number line to round to 10 & 100; use proximity on number line	Use number line & rounding rules to round to nearest 1,000	Use number line & rounding rules for numbers to millions

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (C): Compare and order whole numbers, commonly used fractions, and decimals					
KNOWLEDGE: The student should have the conceptual understanding of:					
1. Models for whole numbers, fractions, and decimals a. base ten blocks b. fraction pieces, pattern blocks, fraction bars, circles, Cuisinaire rods c. money d. graph paper	Match groups of objects using one-to-one correspondence to compare groups of items; use items in ten counting frame; use 2-color counters to model a group with more or less; match group of connecting cubes to correct number	Compare numbers using one-to-one correspondence for groups; order from least to greatest	Compare/order numbers to 1,000; model using number line, 100 chart or place value pieces; build & compare fractions	Compare/order numbers to 10,000; model using number line or place value pieces; building, compare & order decimals	Write value of a digit using multiples of ten; write number as expanded notation; model multiplication of 2-digit number times a 2 or 3 digit number using color tiles & graph paper diagram
2. Models for place value a. base blocks b. place value charts c. Unifix cubes			Model & compare whole numbers & fractions using blocks & pieces	Model whole numbers and decimals using blocks & charts	Model division of a 1 & 2 digit divisor using base ten blocks

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Symbolic place value		Use 100 chart or number line to identify numbers greater than or less than 50; sequence groups of numbers; use connecting cubes to model ten more/ten less	Use number line to identify number before, after or between to 3-digit numbers	Use number line, place value blocks or charts to compare & sequence groups of 10,000 numbers	Use fraction bars, number lines & base ten blocks to compare & order fractions & decimals
The student should be able to: 4. Use symbols: <, >, =		Build trains with connecting cubes to model (4 + 1 = 3 + 2); vocabulary: before, between, after	Use >, <, = to compare numbers to 1,000 & fractions to twelfths; use place value pieces & fraction parts	Write <, >, = to compare numbers to 10,000 and decimals of tenths & hundreds	Write >, <, = to compare fractions, decimals & whole numbers to millions
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the lower base, the student should be able to:					
5. Compare whole numbers, fractions, and decimals to each other with and without models a. compare whole numbers (up to and including seven digits) with and without models	Compare groups of up to 10 items using connecting cubes & diagrams; develop greater than & less than vocabulary	Compare 0-99; group items in tens; check tens number first since tens are greater	Compare numbers to 1,000 using models, pictures, place value mat	Compare numbers to 10,000 using pictures, models & place value chart	Compare numbers to millions using place value chart, Cuisinaire rods to model

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. compare two proper fractions with models		Identify using models which is greater, 1/2 or 1/3	Compare using >, <, =; use fraction pies, bars, etc.	Compare using >, <, =; use pictures, fraction bars, pattern blocks, etc.	Compare using >, <, =; use fraction bars, number line, 2-color chips, etc.
c. compare proper fractions that have the same denominators without models			Use/make a model strategy to solve problems with fractions	Draw conclusions about comparisons of like fractions using fraction models	Model renaming of a mixed number using fraction pieces
d. compare two decimals (up to and including two decimal places) with and without models			Use fraction bars, pie slices & shaded in pictures to compare	Use number line & place value pieces; record equations to compare	Write the decimal for the shaded in part of a base ten block diagram
6. Recognize and determine equivalent forms of a number with and without models a. recognize and determine equivalent forms of a proper fraction with and without models			Use fraction bars, pie slices & shaded parts to model equivalent fractions for 1/2, 1/3, 1/4	Use fraction bars to model; write equivalent fraction equation	Use base ten blocks, Cuisinaire rods, diagrams & place value chart to model decimal fraction equivalents
b. recognize and determine equivalent forms of a decimal (up to and including two decimal places) with and without models				Relate decimals with money; use play coins & place value charts	Relate decimal & fractions to number line & place value chart

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
7. Order whole numbers and decimals with and without models a. order a maximum of four whole numbers (up to and including seven digits) from less to greatest or vice versa	Order numbers 1-10 using counters; develop before, after, between strategy; follow connect the dots 1-10	Order group of 4 double-digit numbers	Use a number line to put sets of 2 or 3-digit numbers in order	Order group of numbers to 10,000 without models	Use number line & place value chart to order numbers to millions
b. order a maximum of four decimals (up to and including two decimal places)				Order groups of decimals using number line	Use place value & number line to order decimals

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (D): Demonstrate an understanding of the meanings of the four basic arithmetic operations through modeling and discussion					
KNOWLEDGE: The student should have the conceptual understanding of:					
1. Meanings of basic arithmetic operations	Model joining situations using everyday events, connecting cubes, real objects; solve problems using act it out strategy; develop vocabulary: add, in all, how many more; model take-away stories using everyday events & counters; develop vocabulary: subtract, less, left	Vocabulary: group, add, increase, join, sum, subtract, decrease, difference Concept: = means "the same as" Addition/Subtraction Facts Development using: pictures, counters, money, real objects, walk up & down big number line; count up/back strategy	Count up or back 1, 2, or 3 to find sums and differences from 20; count on by 1, 10, 01 100 to add (i.e., 65 +3 = 68, 65 + 30 = 95, etc.) Explore multiplication as skip-counting on the number line; explore division concept of equal shares using real objects	Connect multiplication to repeated addition; use arrays & order property to explore multiplication, use zero property & property of one for multiplication; use multiplication table to record facts; multiplication with 0-9; model division as repeated subtraction; use arrays, graph paper & counters; use models to multiplication 2-digit numbers & solve division with remainders	Multiply & divide basic facts using arrays; review basic addition/subtraction facts

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
2. Properties of operations a. zero (additive, multiplicative)		Zero property of addition/subtraction ; model using domino with no dots on 1 side, Demo: 0+5=5+0	Zero property of addition/subtraction ; use as strategy in computation & problem solving	Use zero property to solve multiplication & division facts	Use zero property in multiplication & division
b. one (multiplicative)				Use property of one to solve multiplication & division facts	Use property of one to solve multiplication & division
c. grouping (associative)		Model with cubes for 3 addends – jumps on number line	Investigate grouping strategies to find sum of 3 addends	Use grouping property of additional to solve double digit column addition & to multiply 3 factors; model using arrays	Multiply 3 factors using the grouping property of multiplication; use parenthesis in number sentence
d. order (commutative)		Connect cube trains; turn or flip domino to change order of numbers	Model with ten sticks 2-digit addition of tens numbers (i.e., 50+30=80, 30+50 = __)	Model using rows and columns of arrays; use =as number balance to model	Use to explain balanced equations (i.e. 8 X5 = 5 X 8)

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Inverse operations a. addition and subtraction		Use 2-color connecting cube trains to model subtraction as inverse of addition; Fact Families; write connected facts; use addition to solve related subtraction fact 6 + □ = 9; doubles	Use addition to model subtraction as an inverse operation; use connecting cubes, dominos, make Fact Families with 2-color counters; use inverse relationship of addition/ Subtraction to check 2 & 3 digit computation	Use addition to check big number subtraction; model using place value pieces	Fact review & application
b. multiplication and division			Write story problem using division; solve using different strategies (i.e. repeated addition or subtraction)	Build arrays with color tiles; use array to model rows & columns as groupings; use multiplication fact to solve division fact; use multiply as step in long division	Model with cm. cubes or draw an array to show division as the inverse operation of multiplication; find related facts and missing numeral in equations; check multiplication by using division
4. Relationship between addition and division		Repeated addition; skip count by 2, 3, 4, 5 using models/pictures	Model equal groups of counters; use as repeated addition and multiplication to find how many in all	Connect multiplication, division using models, pictures, arrays, etc.	

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Relationship between subtraction and division			Use repeated subtraction to model division	Model division as grouped subtraction	
The student should be able to: 6. Use models to represent and explain the meaning of each of the four basic arithmetic operations and their properties	Use connecting cubes, 2-color chips, bears, etc. to model addition and subtraction; solve fact problems using picture, coins, drawings	Use 2-color counters to model addition and subtraction facts; match picture to correct number sentence; solve word problems; explore all addition and subtraction combinations for a given number	Use connecting cubes, 2-color chips, tens counting frame to model addition and subtraction facts → 20	Use arrows to show skip counting in multiplication and division; model using connecting cubes, 2-color chips & graph paper arrays; solve number puzzles with mixed +, -, X clues	Use cm cubes & grid paper to model fact arrays; use counters to model division with remainders; model division with remainders & up to 3 digit by 2 digit multiplication with place value blocks
7. Use models to represent and explain inverse operation		Model joining & separating with connecting cubes; build, write fact families using 2-color counters; connect elements of fact families to find related facts, doubles fact families	Model fact families to find sums & differences; use blocks, chips, real objects or drawings	Use multiplication and division facts to create fact family; use △ to model related facts; use counters to model solution for multiplication/division story problem	Use cubes to model multiplication & division as inverse operations

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
8. Use models to represent and explain the relationship between addition and multiplication			Use cubes, blocks, pictures to model equal groups and arrays in multiplication	Use counters to model multiplication as joining of equal-sized groups	
9. Use models to represent and explain the relationship between subtraction and division			Use color chips, counters, real objects to model equal groups of repeated subtraction	Use counters or calculator to model division as repeated subtraction	
PROBLEM SOLVING SKILLS: In problem settings, the student should be able to:					
10. Choose the appropriate operation (s) to solve a problem		Read & solve word problems using pictures, models, counters	Read & solve word problems involving shopping and mixed operation word problems	Use problem solving skill of choosing correct operation to solve word problems; develop definition of each operation using language	Use data from table or graph to solve word problems

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (E): Use and explain paper and pencil procedures for performing whole number calculations					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Number facts a. addition (sums to 18)	Model and draw addition patterns of +1 or more objects; color squares on graph paper to show +1 pattern	Model using pictures, counters, tens counting frame with 2-color chips, addition money amounts, use connecting cube trains. Story problems counting on strategy with number line; use doubles and doubles +1 strategy; use count on 1,2,3; start with greater number and add on; use flash cards	Model with tens frame using make a ten strategy; solve by drawing a picture, counting on number line; use counting on strategies	Use in 3-digit column addition; practice facts	Fact Review

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. subtraction (inverse of addition)	Model and draw subtract one or more objects; illustrate subtraction stories; play connecting cube take-away game; use cross out technique on picture page	Model using pictures, counters, connecting cubes & 2-color chips; use to compare 2 numbers; match story or picture to number sentence; use number line with count back 1,2,3 strategy; use related facts and fact families; use flash cards	Use addition facts to solve subtraction facts; use count back strategy	Practice facts	Fact Review
c. multiplication (up to and including 10 X 10)			Skip count using model & number line; use real objects (i.e., fingers, wheels, etc.) to model multiplication groups; make arrays with 2-color chips; use nickels & dimes for X5, X10	Model using counters, arrays, manipulatives; record on fact table; solve word problems using draw a picture or make a model strategy	Find multiplication rule using input/output table; model 2-digit X 2-digit multiplication using place value blocks & diagrams

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
d. division (inverse of multiplication)			Model division fact using groups of color chips	Model division facts using array pictures, counters, block; relate division to multiplication; use facts and write a number sentence strategy to solve word problems	Model multiplication & division facts using counters & arrays; model & draw arrays modeling division with remainders
2. Place value and regrouping		Model using place value pieces; count & write numbers in correct tens/ones alignment; show difference between 38 and 83	Model tens as groups of ones (2 tens = 20 ones); show a number using color chains, piles of beans & connecting cubes; model regrouping and trading using place value blocks; explain & demonstrate when to group in ones and tens using place value mat; record answers on place value chart; show regrouping and trading changes in problems	Use place value blocks and chart to show addition/ Subtraction with regrouping to 1,000 numbers; build numbers & model trading & regrouping; model double digit multiplication & division with remainders using place value blocks & graph paper arrays	Model multiplication of 3-digit numbers by 2-digit numbers using place value blocks

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Arithmetic computation a. addition (up to and including three, 3-digit addends)		Use counters to show ways to make 10 using 3 digits; add 3 digits as column addition; use 3-color connecting cube trains to demo; add tens & ones in 2 digit numbers; add 10 using 100 chart	Use grouping strategies to find sum of 3 addends; model regrouping using dimes & pennies; show counting on using number line; add 2 & 3 digit numbers; model regrouping of ones/tens; use data from a graph to add 2-digit numbers; use paper & pencil , mental math or calculator to solve	Add up to 1,000 numbers with/without regrouping; add/ subtract dollar amounts	Add up to 100,000 numbers; add/ subtract dollar amounts

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. subtraction (up to and including three digits)		Use number line count back strategy; use related addition fact; subtract tens/ones in 2-digit numbers; subtract using count back strategy on 100 chart	Model trading using connecting cubes & place value blocks; rewrite horizontal number sentence to vertical & record in place value chart to solve; analyze elements of subtraction with regrouping using models; solve using pencil & paper, mental math or calculators; subtract 2 & 3 digit numbers with & without regrouping	Subtract with numbers up to 1,000's with/ without regrouping; subtract dollar amounts	Subtract with numbers to 100,000; subtract dollar amounts
c. Multiplication (up to and including two 2-digit factors)			Model multiplication using counting chip arrays	Multiplication facts; multiply 2-digit numbers using array or counter model	Multiplication facts; multiply 3-digit numbers by 2-digit numbers

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (F): Select and use appropriate whole number computational methods and check the reasonableness of the results					
KNOWLEDGE: The student should have a conceptual understanding of:					
a. number facts b. mental math c. estimation		Use a picture to describe addition or subtraction situation; solve word problems by using various strategies such as write a number sentence, make a model or draw a picture; use domino flip to demo horizontal, vertical writing of +, -	Model the order property & zero property in addition; use doubles and doubles +1 strategy; use addition/ Multiplication table to record facts; model subtracting all or zero from a number; count on or back using number line model; round to nearest 10 or 100 using number line and compare numbers using greater than or less than 50 or 500 when	Explain need for rounding & estimating in addition/subtraction ; use estimate to check the reasonableness of answers; model & explain regrouping in large computations; solve missing digit in mixed operation number sentence; follow multi-step equation and solve; write a fact family (X, ÷) given 3	Explain break apart, compatible numbers or make a ten strategy using place value blocks & diagrams; write explanation of solution for missing number in a mixed equation; explain & follow steps for long division

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
		add to solve a magic square; use count back strategy to complete subtraction fact table; use related facts to solve; count on & count back strategy	rounding; add/ subtract 10,20,30 or 100,200,300, etc. using mental math	numbers; explain how to use X to solve ÷; find which number sentence does not belong in a fact family; choose the correct number sentence to solve a word problem	
1. Variety of mental computation strategies					
2. Calculator use		Skip counting using the constant; use to find sums & differences	Use to count on, find missing addend & add/subtract 2 & 3 digit numbers	Use calculator when numbers are too large or too many in problem solving; find product in 2-digit multiplication	Use calculator to find solutions using all operations
3. Paper and pencil procedures		Write solutions to computations & word problems; write addition & subtraction facts in horizontal & vertical form; write appropriate symbols +,-,=; write all appropriate numbers &	Write missing numbers in addition, subtraction, multiplication, and division facts; record facts on table, write number sentences & computation to solve word problems; use	Write solutions to addition/subtraction number and word problems with numbers to 1,000's; record column addition answers; write 2-digit multiplication problems & long	Write solutions to addition/subtraction problems with numbers to 100,000 using various strategies including partial sum and difference; write solutions for large

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
		symbols; write answers on table or chart; circle the sum of the first 2 numbers when solving 3 number column addition; draw lines to show jumps on the number line; record numbers in tens/ones chart	models to tell how many hundreds, tens, ones in number; write numbers in place value chart to add and subtract; write regroup/trade numbers correctly; record data on chart or table	division problems using correct numbers & signs; use place value chart to record & solve computation	number multiplication using partial product, grid paper & lattice multiplication; solve long division using grid paper
The student should be able to: 4. Demonstrate proficiency with number facts using a variety of fact strategies	Model addition and subtraction facts using models, pictures, counters; write number sentences for addition/ subtraction problems with combinations to 6	Model facts using pictures, drawings, real objects, counters, tens counting frame, connecting cubes, dominos, coins, 2-color chips, number line & flash cards; solve facts by using count up, count back, doubles, doubles +1 or −1 and fact family strategies (combinations to 20)	Model facts using connecting cubes, floor number line, mental math count up or count back; place counters on tens frame to model groups of ten strategy; use draw a picture strategy; make and use flashcards for combinations to 18	Record addition/ subtraction facts in vertical and horizontal form; model multiplication & division facts using pictures, counters, cubes; record multiplication/ division facts in vertical and horizontal form; use facts to solve word problems; apply facts in solving large number, mixed operations and computation	Solve mixed addition/subtraction facts as review

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
5. Select, use, and explain an appropriate computational method (mental math, paper and pencil, estimation, calculator) to solve a problem		Use calculator to solve facts & find large sums	Model estimating & rounding, counting on & back using number line; use calculator to solve 2 & 3-digit numbers	Model large addition problems using expanded notation equations; use associative property to solve subtraction problems with regrouping	Determine appropriate method by looking at the numbers; find a compatible number to estimate an answer
6. Determine the reasonableness of an answer		Determine close estimate using real objects; use rounded numbers as appropriate estimate; estimate time, length & weight	Estimate amounts in word problems to find answer that makes sense; use make an estimate strategy with measurement	Look for information in word problems to tell if an answer makes sense; guess & check problem solving strategy; estimate groups of objects using models & numbers	Determine whether you need an exact answer or an estimate

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (G): Perform operations with commonly used fractions and decimals, using models					
KNOWLEDGE: The student should be able to:					
1. Model fraction addition and fraction subtraction				Use fraction bars to model addition and subtraction of like fractions; identify shaded parts in picture	Use fraction bars & number line to model addition/subtraction of unlike fractions
2. Model decimal addition and decimal subtraction			Add and subtract dollar amounts using dollar signs & decimal points	Write decimal point and dollar sign for addition/ subtraction problems; write problem and solution for addition/ subtraction word problem	Write the decimal and fraction shown by a graph paper or number line model
3. Model decimal multiplication and decimal division involving one whole number ad one decimal				Multiply money amounts using decimal notation	Solve multiplication and division of money using decimal notation

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER I: NUMBER SENSE, OPERATIONS, AND PROPERTIES

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (H): Count and perform simple operations using money					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Coins and paper money and their corresponding values	Identify and sort all coins; use play money to identify heads or tails	Count and show coin combinations to $1.00 using 1¢, 5¢, 10¢, 25¢; use play money and shopping activities	Count coins in sequence; show regrouping dimes and pennies; know value of all coins and how to group count them	Model equivalent sets of money using coins & bills; compare pictured amounts of money	
The student should be able to:					
2. Find the value of a collection of coins and paper money	Know real coin values to 25¢; count mixed group of coins with total up to 20¢; count on pennies to a nickel or to a dime	Identify 1¢, 5¢, 10¢, 25¢; coins; count in like and mixed groups; use act it out strategy to solve shopping problems	Model and count collection of coins; show same amount using fewer coins	Make a table to show equivalent amounts of money; solve word problems with various mixed amounts of coins & paper money	Give total value of mixed group of coins and bills

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Solve problems involving money a. determine total cost	Solve addition problems by using pennies; count and compare groups of pennies	Solve story problems using make a model strategy; use shopping cards or games	Determine prices for various items and draw coins/bills to show amounts equal to the price	Solve computation and word problems using addition of money amounts	Solve word problems using cost data from a table or graph
b. determine change			Count on from price to determine change; model using play money or picture	Count on to determine change; write and solve subtraction problems with money amounts	Solve word problems using item costs with change
c. determine different ways to represent money values		Match price with various amounts of coins; show amounts in various ways	Model amounts of money using fewest coins; show 2 ways to make the same amount of money using play coins or drawing	Use data from a sign or menu to compute cost and change; compute ticket costs; list and chart combinations for amounts of money; relate decimals and money	

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER II: MEASUREMENT

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (A): Select and use appropriate nonstandard and standard units of measure to describe, compare, and order various quantities					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Measurable attributes a. length, distance, area, volume	Compare real objects using vocabulary: Longer, shorter, taller, heavier, lighter	Compare shortest/ longest length; measure inch, foot, yard	Measure length using inch, cm, foot, meter, yard; pound, ounce, gram, kg., temperature	Measure length to nearest inch and ½ inch; measure area and perimeter of a shape in cubic units	Measure length to nearest 1/2, ¼ or 1/8 of an inch; explore relationshi p between area perimeter of plane figures and solids

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. capacity, weight, time, temperature	Compare real objects using vocabulary more than, less than, most, lest; tell time to hour on analog and digital clock	Compare heaviest/ lightest using balance scale; compare size of fractional pieces; measure time using calendar; tell time to hour and ½ hour; lapsed time of 1 or ½ an hour	Tell time to the nearest minute; use a calendar; determine how many cups fill a pint and a quart; measure mass using gram and kilogram; read °F	Tell time to the nearest minute; use a calendar to determine lapsed time; estimate and weigh using ounces, pounds, grams and kilograms; model mass using objects on balance scale	Read and write time to nearest second; find capacity of units using cup, pint, quart, gallon; find mass using gram and kg and ml or liter to find capacity
2. Nonstandard units of measure	Measure using crayon and connecting cubes to measure and compare height and weight of objects	Measure length using paper clips, beads, string; measure weight using connecting cubes	Measure length using paper clips	Choose appropriate unit of measure for length and weight	Choose appropriate unit of measure for length and mass
3. Standard units of measure a. all customary units (excluding bushel and peck)	Compare length of items to a 6in. ruler	Find number of cups that equal a pint, quart and gallon; measure using in, ft. cm	Measure and weigh real objects using inch, foot, cup, pint, quart, ounce, pound	Measure and weight real objects using inch, foot, ounce, pound, cup, pint, gallon	Measure and weight real objects using inch, foot, yard, mile, ounce, pound, cup, pint, gallon
b. all metric units (excluding deka- and hecto-)		Measure cm length to 10 cm	Measure length in inch, ft, cm, measure mass in gram or kilogram	Measure and weight real objects using centimeter, meter, gram, kilogram	Measure using centimeter, meter, decimeter, gram and kilogram

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
4. Personal referents that approximate standard units of measure		Use 2 fingers next to each other as 1 inch; use 1 finger as 1cm	Use finger width as 1 cm	Use hand span to measure a decimeter, arm span = 1 meter; kilometer = little more than ½ a mile	Use distance from first knuckle on thumb to the end to measure 1 inch; 1 ft. about the length of a piece of paper; 1 yard about the length of a baseball bat
The student should be able to:					
5. Recognize the need for a standard unit of measure		Compare size of non-standard units	Explain how measurement would differ using inch or cm; identify appropriate measuring tool	Explain choice of unit of measure for real objects	Explain need for standard units as differences in size of people measuring
6. Use rulers, scales, thermometers, and clocks to accurately measure length, weight, temperature, and time	Use 6 in rule line to compare length; sequence events of th4e day using series of clocks; write	Measure length of real objects; use measuring cups; match analog and digital times to hour and ½ hour	Distinguish between AM and PM when telling times; understand time relationships for length of activity; measure	Record elapsed time on schedule; write digital time to minute; measure temperature using °F and °C	Use schedule data to find lapsed time; write digital time using story problem; measure length and distance; read
	the hour shown on a digital or analog clock		accurately using ruler; use pint, quart, cup, liter containers to measure capacity		temperature in °F and °C

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
7. Select the appropriate unit of measure for length, weight, capacity, temperature, area, and volume a. customary b. metric			Compare standard units of measure to paperclip (1 in) or hammer (1 ft); compare 1 slice of bread (1oz) or 1 loaf of bread (1 lb) for weight	Compare standard units of measure; estimate length and weight of objects; use cm cubes to solve for area and volume	Identify tools to use for linear measurement; find combinations of containers to find capacity; estimate/measure length of real objects
8. Select the appropriate unit for measuring time	Tell time to the hour using analog and digital clock models	Compare how long it takes to do things (i.e., less or more than a minute) Use minute, hour, week; tell time to hour and ½ hour	Tell time to 5 minute interval on analog clock; estimate how long an activity will take	Model telling time to the minute with play clock; write accompanying digital time	Read and write time to nearest minute or second using digital form
9. Select the appropriate measurement tool for a given situation	Use vocabulary including hour and o'clock to tell time	Measure using clock, ruler, yardstick, cups, balance scale	Choose inch, foot, centimeter, or meter to measure height and length; use Farenheight thermometer	Estimate length in inches and centimeters; check with ruler; estimate temperature, check °C and °F with thermometer	Estimate length or real objects; choose most appropriate unit of measurement for real objects

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
10. Use personal referents to approximate measures	Measure by hands or feet length to estimate and measure objects	Compare lengths by touch; make a paper ruler; use connecting cubes to approximate length and weight	Compare length using paper clips; estimate and measure length in centimeters and meters; compare and estimate capacity in liters and weight in pounds/ ounces	Compare lengths; width of paperclip is about 1cm; length of paperclip is about 1 in.; compare mass: 1 paperclip = about 1 gram, a large book = about 1kg	Compare capacity of containers using eye dropper and soda bottle; choose approximate measure based on size of container
11. Solve problems involving measurement a. perimeter of a polygon			Build shape on geoboard; use ruler to measure length of sides; develop vocabulary: perimeter	Build polygon using color tiles; write on graph paper	Build polygons; write a formula to solve for perimeter
b. area of a rectangle by grid or given formula				Build plane figures using color tiles; write on cm graph paper	Use diagram and dimensions to find area of plan figures; follow formula
c. area of other polygons by grid				Build polygons using color tiles; write and measure on cm graph paper	Use centimeter cubes and graph paper diagrams for plane figures
d. area of irregular regions by grid					Model color tiles and use diagrams to solve for irregular shapes on grid

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
e. volume of a rectangular solid by counting cubes				Model volume using connecting cubes as 1 cubic unit; use 3D pictures of shapes	Build using cubic units; follow formula for volume
f. time	Make and analog clock with movable hands; recognize forward movement of hands in a series of clock pictures	Identify parts of an analog clock; show 30 minutes later from o'clock; use a schedule to get information to tell start time and end time	Draw the hands onto the clock to show the time; write the time to 5 minute intervals; identify AM/PM using picture or story	Use start time & length of activity to find end time; model lapsed time on manipulative clock; solve word problems using lapsed time	Calculate elapsed time using data from clock and schedules; use movement of clock hands and subtraction to solve
g. temperature				Compare temperature on weather chart; use picture clue to choose best temperature estimate	Estimate change in temperature and approximate temperature for a specific item or activity

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
h. liquid capacity		Use cup, pint, quart, gallon units to compare capacity	Determine how many cups = 1 qt. and 1 pt. Using plastic measuring cups	Estimate and measure capacity; model liquid measure equivalencies; use table of measure for standard and metric	Estimate need for capacity of real objects; use make a model, act it out or write an equation to solve
i. weight			Weigh real objects on balance scale in pound, ounce, gram or kilogram	Weigh real objects using balance scale	Weight real objects using gram or kilogram on balance scale
12. Compare and order objects according to a measurable attribute	Use 1 crayon, 6in., or a specific number of connecting cubes as reference point; use real containers to compare capacity and weight	Put objects in order according to length, weight, and capacity. Use real objects and measuring tools	Compare and estimate capacity in liters using plastic/real containers	Compare weights of objects to 1oz. or 1lb., 1g. or 1kg.	Compare small objects to fraction of an inch linear measure-ment

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER III: SPATIAL SENSE AND GEOMETRY

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (A): Describe and use the properties and relationships of 2 and 3-dimensional shapes					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Basic geometric elements a. point, line, line segment, ray b. intersect, parallel, perpendicular				Identify lines, line segments and rays; model on graph paper; compare angles to right angle	Identify lines, angles & rays on a diagram; model using diagram and blueprints
2. Standard notations: (line segment) AB, (ray) AB, (line) AB, <ABC					Identify and name points, lines, line segments and rays
3. Basic 2-dimensional geometric terms, shapes, parts of shapes, and angles a. polygon, triangle, quadrilateral, square, parallelogram, rectangle, trapezoid, rhombus, pentagon, hexagon, octagon, decagon	Identify plane shapes as part of a larger figure; recognize shapes with line of symmetry	Identify plane and solid figures; compare to real objects	Identify plane and solid figures; sort and classify plane shapes	Identify, describe and classify polygons, triangles, quadrilaterals	Identify equilateral, isosceles and scalene triangles using diagrams; model on geoboard

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. side, vertex, diagonal	Identify attributes of pattern blocks	Identify sides and corners of a shape	Identify and count sides/corners of a shape	Identify, describe and classify polygons, triangles, quadrilaterals	Identify sides and points of a polygon; make ven diagram to sort and order
c. circle, diameter, radius	Identify circle as a plane figure	Identify and name shape	Identify and name shape	Draw circles and identify their parts: center, diameter, radius	Identify center, chord, diameter and radius using diagram and model
d. angle, right, acute, obtuse, straight, vertex				Identify right angle; compare less than or greater than a right angle	Identify right, acute, and obtuse angles; determine how line segments relate to a line
4. Basic 3-dimensional geometric terms, shapes, and parts of shapes a. sphere, cube, cone, pyramid, rectangular prism, cylinder	Names and identify sphere, cone, cube and cylinder	Name and identify shape	Name and identify all shapes	Describe properties of solid figures; name the faces that make up solid figures	Identify solid figures by their dimensions
b. edge, face, vertex, base	Identify shape of face or base using 3-D object and attrilink	Sort and classify solids with flat surfaces; build models; count sides and corners of plane figures	Make plane shapes from the faces of solid figures using wood 3-D shapes	Use solid wood shapes, real objects and diagrams to label face, edge, vertex	Use solid figures and diagrams to find faces, edges and vertices

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Two-dimensional properties a. angle classification b. number of sides c. curves or line segments	Identify and sort plane shapes by attributes	Write names for 2 and 3 dimensional shapes	Identify and count sides of plane figure	Classify figures by number of line segments, angles, right angles	Classify figures by number of congruent sides and size of angles
6. Three-dimensional properties a. number of faces, edges, vertices		Draw shapes on cheesebox paper; draw both shapes with same dimensions	Match plane figure to face of a solid figure	Name solid figures; trace faces to classify; use diagram	Find plane figures in the faces of solid figures; use everyday objects
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
7. Use properties, definitions, and relationships to identify, classify, and describe 2-dimensional geometric shapes a. number of sides b. classification of angles c. curves, line segments	Sort/order pattern blocks and attrilinks	Sort/order plane figure shape pieces; make shapes using holding hands activity; combine shapes using pattern blocks	Sort/classify plane shapes; look at multi-shape designs and count how many shapes are inside	Compare number of sides and angles to classify polygons and quadrilaterals; explain why a circle is not a polygon; describe right angles, line of symmetry, congruent figures and parallel lines	Compare angles and number of sides using geoboards; order using Venn diagram to organize date

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
8. Use properties, definitions, and relationships to identify, classify, and describe 3-dimensional geometric shapes a. number of faces, edges, vertices	Identify and match solid figures	Draw shapes; find real objects with same shapes; find pictures in magazines to classify	Count number of faces on real 3-D objects	Describe and name 3-dimensional shapes using number of faces, edges and vertices; use diagrams and real objects	
9. Use manipulatives such as pattern blocks, geoboards, tangrams, solids, and paper to demonstrate properties of 2-dimensional and 3-dimensional shapes	Use attribute links to identify similarities (i.e., size, shape, color)	Identify shapes that will stack, roll or slide; count number of flat surfaces on a shape; find line of symmetry; use geoboards, wood solids, pattern blocks	Use attribute links, geoboards, pattern blocks, wood solids and cheesbox paper to model shapes	Model solid figures using real objects; trace and fold paper to build pyramid, cube, etc.; use pattern blocks to model congruent figures; model slide, flip and turn on dot paper	Model solids using diagrams and wooden blocks; show rotational and line symmetry using pattern blocks and geoboards; building and measure angles on geoboard

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER III: SPATIAL SENSE AND GEOMETRY

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO: (B) Use coordinates and paths in maps, tables, and grids.					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Methods of locating points in the plane a. point, coordinates, ordered pair b. horizontal, vertical, latitude, longitude c. scale, table, grid	Identify left and right; distinguish left and right side of path	Follow directions on grid map; use positional terms; use draw a picture or make a model strategy	Use points on a map to show locations; then, measure distance between points	Locate points on a grid using coordinate number pairs; use horizontal and vertical lines on the grid	Locate points on a coordinate grid; Use X and Y axis and X & Y coordinates when naming ordered pairs
The student should be able to:					
2. Identify the coordinates of a point on a grid or map		Plot points on a grid map	Identify objects at a point on a grid	Identify ordered pairs used to locate point on grid	Name ordered pairs using X and Y axis
3. Plot a point given its coordinates		Draw arrows on grid to show directional movement; count blocks moving in all directions	Count blocks and move on grid	Explain how to find a point on the grid	Graph series of coordinates from frequency table on grid

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
4. Construct a simple figure by connecting points given a series of ordered pairs		Connect dots on cheese box grid to form shapes; identify congruent shapes when turned	Identify shape on grid after being turned or flipped	Locate an object on the grid, then give ordered number pair to describe its location	Locate points on the grid; connect to follow trail or create shape
5. Use data from a table or grid to solve problems		Follow directions for movement on grid map to find location	Use data on a table to answer questions and solve word problems	Compare information on table; use information to answer word problems; use to group data; use tally chart and frequency table	Solve problems using data from table or grid wit prices, temperatures, times, distance, etc.
6. Interpret information from a map, grid, or table	Represent a path and describe it, using directional terms	Use directional vocabulary to identify position; describe position of an object following a designated path	Use map information and data on table to solve word problems	Compare and contract table data, use data to find differences; interpret date from a survey; measure distances on a map; solve word problems using table information	Compare and combine table data; group and analyze data; estimate to solve

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
7. Construct a grid or table using data				Make a table to record coin combinations; record lapsed time data on table; use "make a table" strategy in problem solving	Use frequency table data to identify relationships and create plot coordinates on grid

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER III: SPATIAL SENSE AND GEOMETRY

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (C) : Use transformations to determine spatial relationships among 2-dimensional figures					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Geometric transformations (e.g., flip, turn, and slide) and related properties a. flip, turn, slide, stretch, shrink	Identify plane shapes in different orientations; model by turning pattern blocks	Identify turn in a diagram; model using pattern blocks	Identify flip, turn, slide in a diagram; model using plastic shapes	Identify in a diagram; model using pattern blocks, cheese box paper and real objects	Draw on grid; build on geoboard; model using pattern pieces
b. Congruence, similarity, symmetry	Sort/classify circles, squares, triangles, rectangles by shape; identify real items that are in shape	Sort and order objects by size and shape; build the draw shapes on cheese box paper; use pattern to model congruency	Sort similar shapes using real objects and a diagram; use cheese box paper to draw shape and line of symmetry	Fold and draw lines of symmetry on real objects and on diagrams	Fold, draw, cut, compare shapes

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
c. tessellation				Combine plane figures to form patterns	Build and compare using pattern blocks, and Pentominoes; trace and color
The student should be able to: Identify transformations in nature, art, and other areas	Identify "moving" shape as a turn	Make a linear pattern by turning one pattern block	Make a linear pattern by flipping and turning one pattern block	Model using pattern blocks and stained glass window, paintings, etc.	Model using pattern blocks and kaleidoscope, building design, butterflies, etc.
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the should be able to:					
3. Determine lines of symmetry	Fold and color to show 2 halves	Model by fold and tract activity; use pattern blocks; mirror image	Construct symmetrical shapes on cheese box paper	Fold, trace, draw on various real objects and shapes; identify number of lines of symmetry on a shape	Build and draw designs on graph paper and dot paper; identify number of lines of symmetry
4. Identify congruent figures		Model on grid paper or geoboards	Name the 2 parts (halves) of a plane figure as being congruent	Find and build congruent shapes using pattern blocks on dot paper	Demon-strate transformations and model using irregular shapes on dot paper

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Use transformation to draw the image of a figure	Trace blocks and attrilinks, then turn, to model different orientation	Model using reflection line	Demonstrate how a flip is different from a turn by using manipulatives	Draw figures on dot paper to model movements	Draw figures on dot paper to model movements
6. Identify the transformation used, given a figure and its image	Describe turn as "moving" the shape		Identify movement of shape as flip, turn or slide	Use arrows to show direction of movement	Draw figures on dot paper to model movements
7. Classify a variety of 2-dimensional shapes by congruence and/or similarity	Sort and order shapes in groups using color and shape; use attrilinks and pictures	Sort and order objects including bears, buttons, shapes, etc.	Identify circles, squares, triangles, rectangles, and ovals; demonstrate line of symmetry	Model using drawings and pattern blocks; build on 1in. graph paper using color tiles	Model using drawings and geoboard; compare figures to determine similar or congruent

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER III: SPATIAL SENSE AND GEOMETRY

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (D): Demonstrate the ways in which geometric shapes and objects can be measured, combined, subdivided, and changed.					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Measure of shapes a. Area, perimeter, volume			Construct and draw 2-figures with same perimeter using geoboard or dot paper	Construct and draw figures on graph paper with specific perimeter using color tiles as model	Identify polygons and find the perimeter by adding the lengths of the sides
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the students should be able to:					
2. Predict the resulting shape when combining, subdividing, and changing figures	Stretch rubber band to form triangle, square, etc.	Model by building or tracing sides-circle angles	Model using 2 squares or 2 triangles to make a rectangle with pattern blocks	Model shapes and figures using color tiles on 1 in. graph paper; count square units inside; analyze 3-dimensional diagrams	Model shape on geoboard, then, enlarge by multiplying each side by 2; make a model on graph paper, then enlarge on poster

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Estimate the area of a region by identifying familiar subdivisions				Identify solid figures used to make an object; use diagram	Use tangram shapes to build design
4. Determine the perimeter of a polygon made up of unit squares			Construct figures with given perimeter on geoboard	Use color tiles to estimate number needed to go around outside edge; use graph paper diagrams	Use centimeter grid paper to diagram plane figures
5. Find possible perimeters for a given area made up of unit squares and vice versa				Solve word problems using make a model strategy; explain how to find perimeter of a figure when given only 2 measurements	Identify and construct plane figures on grid paper using centimeter units
6. Determine the number of cubes in solid using model			Use make a model strategy to count number of cubes in a given shape	Model solid figures using connecting cubes; build shapes in diagram; use to determine volume	Build using centimeter cubes; use diagram to model real objects

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER IV: DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (A): Determine the probability of a simple event and predict outcomes					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Probability and related vocabulary a. event, chance, outcome, fairness, randomness		Game spinner activities	Record tally marks for coin toss experiment and grab bag of color tiles on tally table	Investigate using coin toss, spinner, and dice games; define what makes a fair game; record list of outcomes	Predict outcomes of an event; record outcomes on table
b. possible, impossible, certain, probable, equally likely, fair, fairness, chance		Use spinner cut in ¼ shaded in ¾ or ½ to compare	Determine what color tile will probably be chosen most often	Determine chance of events as possible or impossible by analyzing models: spinners, objects in paper bag; create spinner to produce specific outcomes	Chart possible outcomes using tree diagram; determine likely or unlikely outcome and equally likely chance of an event
The student should be able to: 1. Determine the probability of simple events		Predict outcome of spin using two different spinners	Predict outcome of color tiles drawn from bag	Identify, predict and record outcomes using data from a line graph; record outcomes on tally table	Compare spinners and predict probability; write the probability of a simple event as a fraction

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
1. Solve problems that involve predicting the outcome of a chance event		Record spin data on chart or table using tally marks	Use tally data to solve word problems	Use data to predict outcome of experiment; name outcomes as likely or unlikely	Predict outcomes and compare with actual events; determine fairness based on spinner configuration or variety of a group of items

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER IV: DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (B): Collect, organize, analyze, and interpret data					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Measure of central tendency and descriptions of sets of data a. Mean, median, mode, average				Develop concept and understanding of mode and range using diagram	Develop concept of median, mode, mean and coverage; use algebraic expression to solve
b. frequency, data, survey, range, tally	Build and identify 1-25 using tally marks; use to record data	Record data on tally chart, picture graph, vertical/ horizontal bar graph or table	Make and interpret 2 and 3 category bar graphs and tally tables	Identify use of line graph, tally table and frequency tables in displaying data	Develop line plot to record data; find range and outlier for given data
2. Displays of data a. frequency tables	Record tally marks on a tally table; record counting data	Show data using tally marks and numbers	Collect and organize information on a tally table	Develop uses for displaying survey results on frequency tables; use to record game scores	Display experiment results on frequency tables; use data in problem solving

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
b. circle graphs, pictographs, bar graphs, broken line graphs	Show one-to-one correspondence using objects on a multi-rowed grid; present in horizontal or vertical position	Draw pictures, record times color bars, write numbers on graphs	Make and interpret picture graphs, vertical and horizontal bar graphs	Identify and use key on pictograph; use vertical and horizontal bar graph; interpret tally data to line plot	Compare data on bar graphs; interpret frequency table to bar graph and line graph; organize and interpret data in stem and leaf plots
PROBLEM SOLVING SKILLS: In problem settings using abilities that comprise the power base, the student should be able to:					
3. Represent data using an appropriate graph	Display data on a multi-row grid with real objects or pictures; color grid boxes to create bar graph	Display data using drawings and numbers	Collect and display data on 2 pictographs using different scales	Record data on pictograph, bar graph and line plot; write title and labels; decide best scale to use	Read and organize data on line plot and in stem and leaf plot; compare to bar graph and frequency table; make and read double bar graphs
4. Analyze data using mean, median, mode, range, and frequency	Use tally table to find solutions to questions	Use data to answer appropriate questions; use table to look for number patterns	Use information from graphs to solve addition and subtract problems	Explain how a line plot shows data; describe the scale; find mode and range on line plot; use to show activities and survey results and temperatures	Explain a different uses for types of graphs; describe use of key in double bar graph; explain use of link graph to show data changing over time

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Make inferences based on organized data and displays	Count objects, the color in square on graph paper; use to answer questions about the data	Find fewest/ greatest amounts; use data to solve addition/ subtraction problems and follow counting patterns (i.e., add 2 more every day)	Use bar graph to show comparisons; identify least and greatest; analyze data in graphical form to develop problem solving and thinking skills	Use data to find and extend patterns; use graphic aids to compare, analyze, and find solutions to word problems	Use data to identify trends and describe probable causes; draw conclusions based on data and describe who might use particular types of graphs

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER IV: DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (C): Follow, devise, and describe algorithms					
KNOWLEDGE: The student should be able to:					
1. Follow an algorithm to accomplish a given task	Give directions and visually represent a path; model using real objects on a map	Model facts using manipulatives; follow rule to complete function tables; count and move spaces on a grid map	Record and use data on frequency table to find a pattern and model adding on in addition	Use data on table to find rule and write accompanying number sentence; use rule to extend and solve	Write equation to show frequency table rule; use function table data to write equation and solve for variables
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
2. Develop an algorithm	Follow and describe movement on a path using words left and right	Solve story problems using write a number sentence strategy	Explain counting procedure used to find number of cubes in a shape	Use data from table to write appropriate number sentence solution	Use data to solve equation with 2 variables
3. Describe a set of instructions to accomplish a given task	Give and follow directions on a path; use blocks, toys, classroom objects	Use ordinal numbers to give step-by-step instructions; show position of hands on the clock for a particular time	Give and follow oral instructions to build a 3-dimensional shape using connecting cubes	Follow directions to fold and find line symmetry, make a bar graph, model data using counters	Follow directions to transfer data to appropriate graph; use, understand, plan, solve, check procedure in problem solving

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER IV: DATA ANALYSIS, PROBABILITY, AND DISCRETE MATHEMATICS

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (D): Devise and use strategies for puzzles, games, and counting problems, as well as problems involving combinations and permutations.					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Network					Use plot diagram to follow paths
2. Tree diagram					Find number of possible outcomes by making a tree diagram
3. Make an organized list for simple counting problems			Make a list of coin combinations using chart; use t-chart to model multi word problems	Use t-chart or table to solve word problems with patterns	Make a list to display possible outcome combinations
4. Make an organized list of all possible combinations of permutations for a simple solution			Use models to show combinations (i.e., pizza toppings)	Use models and record combinations as a diagram	Make a list to determine outcome

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
5. Draw and interpret networks and tree diagrams					Explain meaning of represented data and importance of ordering data
6. Compare and evaluate different strategies for playing a game	Investigate path games, matching games, spinner games, tic-tac-toe	Have experience playing pair games using dice, cards, spinners	Have experience playing games with dice, spinners, cross number puzzles and mazes	Analyze steps in game strategy; explain changes caused by altered choice of moves	Solve number riddles and explain strategy; plan random number generated games
7. Devise and apply strategies for solving puzzles and games	Recognize result of moves on different spaces of game board; experience change of spinner games	Solve cross number puzzles, magic squares, addition/ subtraction bingo	Use alphabet code to solve mystery answer using +, - problems	Identify winning moves in games, predict outcomes and record results of chance games	Identify logical reasoning needed for number riddles and cross-number puzzles

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER V: PATTERNS AND ALGEBRA

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (A): Use a variety of materials to extend, create, and describe patterns, sequences, and relationships that are in mathematics and other disciplines.					
KNOWLEDGE: The student should be able to:					
1. Recognize patterns, sequences, and relationships including those found in nature and art	Identify color, shape, design patterns; model using real objects: students, links, seashells, etc.	Show patterns on the number line and 100 chart; use 2-color chips , connecting cubes to model patterns; write missing numbers in sequence; identify, describe and extend patterns with manipulatives	Skid count on number line and 100 chart; use frequency table to identify patterns; continue pattern in numerical sequence; make and continue t linear shape pattern	Identify skip counting, growing patterns, decreasing patterns; use to model multiplication and division concepts, compute cost; continue shapes and number patterns	Use basic facts and number patterns to multiply mentally (i.e., times 10, 100, 1000) and find large quotients (i.e., $720 \div 9 = 80$); find pattern of growth in area and perimeter based on size of shape
2. Recognize that most patterns and sequences continue indefinitely	Model continuing pattern using bottle caps, counter chips, etc. on strips; extend around classroom	Circle/color patterns on 100 chart; extend patterns using color and shape	Count on or back following rule with numbers to 1,000 using mental math	Write number sequences as a series; extend using counting pattern or rule	Extend multiplication and division patterns to larger numbers; extend counting patterns above and below zero on number line

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Use concrete, pictorial, and symbolic representations to model patterns and relationships	Model patterns using real objects and pictures	Model pattern extensions using pattern blocks, connecting cubes and real objects	Use 100 chart to model skid counting; draw clock hands to model quarter hour time pattern	Recognize a rue when written in linear form on a table; use basic facts and place value patterns to multiply and divide multiples of 10, 100, and 1,000	Extend growth patterns using color tiles to illustrate; compare shape and size to continue 3 part pattern
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
4. Reproduce and extend numerical and non numerical patterns	Solve problem using find a pattern strategy	Model counting patterns using macaroni sticks; tens/ones place value pieces, 2-color chips, pattern blocks, etc.	Identify which group of numbers is following a counting rule; use pattern block to model flip/turn shape pattern	Use number line, multiplication table, data table to identify rule and extend pattern; model shape patterns using pattern blocks and graph paper designs	Explain how you can use patterns for big multiplication with multiples of 10; make a model to find a pattern for bead, shape and number patterns; describe pattern changes
5. Create numerical and non numerical patterns	Create pattern using real objects	Put together repeating units to make a pattern; use manipulatives	Create and extend a linear pattern by flipping and turning	Build and extend patterns using pattern blocks	Build and extend patterns using beads, dates, shapes

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
6. Describe numerical and non numerical patterns	Name patterns by saying name or each object in pattern	Skip count on number line and 100 chart; observe design patterns on 100 chart; use problem solving strategy to find pattern to solve	Model 2, 5, and 10 skip counting using 2-color chips, connecting cubes, coins and real objects on graph paper and dot paper	Explain ordering and comparing numbers, trading and regrouping in computation, finding elapsed time, rules for patterns, describe plane figure movements, rules for rounding and estimating	Explain procedure pattern used in long division; describe counting patterns on calendars; identify missing numbers in patterns; observe and describe growing patterns

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER V: PATTERNS AND ALGEBRA

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (B): Use algebraic concepts and processes to form, describe, and verify generalizations based on observations of patterns and relationships					
KNOWLEDGE: The student should have a conceptual understanding of:					
1. Basic algebraic elements a. variable, expression, open sentence, rule		Match model with number sentence using connecting cubes; solve word problem by writing number sentence; find missing addend in addition problem; match place value pieces with appropriate number sentence	Find missing number in addition or subtraction number sentences using color chips; use tens counting frame and place value pieces to model double digit addition/ subtraction	Solve for missing number in facts and computation number sentences; find missing factor in multiplication and divisor in division	Write an equation with variables to find a frequency rule, identify order of operation using parentheses; write input/output rule as variable equation

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
The student should be able to: 2. Construct an open sentence when given a rule		Write number sentence for constructed model; use picture as model to write number sentence; use crossing out of picture to show subtraction number sent; circle and add the greater number first in addition	Draw a picture or make a model to solve addition/ subtraction word problems; record numbers and symbols for word problem solutions; find a pattern in multiplication and use to find missing multiples	Write number sentence to solve word problems using mixed operations, multiplication of 3 digits, long division; model using color chips; solve algebraic equation and solve for variable	Write an equation or expression to solve for unknown; choose equation needed to solve word problems
3. Construct a graph from a table		Record tally marks, then give total number; make a bar graph that matches the tally marks; make a bar graph to solve a problem	Collect data and display it on a picture graph or multi-category bar graph	Use collected data to construct pictograph, tally table and bar graph	Use frequency table data to make vertical or horizontal bar graph with key
PROBLEM SOLVING SKILLS: In problem settings using abilities that comprise the power base, the student should be able to:					
4. Create a rule based on input and output data		Use addition/ subtraction rules to complete function tables; develop term "rule"	Interpret and use skip count rule to complete multiplication frequency tables	Use linear table data to identify rule and extend pattern	Find and write rules for equations with variables using data from function table

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
5. Solve a simple open sentence involving one operation		Record in vertical or horizontal form, all numbers in problem them, solve; find missing addend; find missing number in fact family set	Solve addition/ subtraction facts with combinations to 20; find missing numbers in problem using all operations; write complete addition/subtraction fact family given 3 numbers of family	Solve addition and subtraction number sentences; use inverse operation to check; multiply 3 factors using Associative Property of Multiplication; find quotient in short division	Solve addition/ subtraction number sentences; write a whole number as a product of 2 factors; find sums and differences with decimals
6. Translate among tables, rules, open sentences, graphs		Use clues and reasoning to find missing number; complete +, - function tables; count on to find missing addend in subtraction; use doubles and doubles +1 strategy to solve	Look for number patterns on multiplication tables to solve; identify and extend rule by completing table; us mental math to add 100, 200, etc.	Use problem solving strategy of make a table to solve problems; use to record coin combinations, making change and time schedules; find	Explain use of specific table or graph; describe how to place data for easy use; predict future outcomes based on rule from frequency table;
			to a number and record on addition table	data on table for multiplication, length, volume, temperature	compare data generated from rules using different operations

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
7. Form, describe, and verify generalizations based on observations of patterns and relationships	Solve problem using find a pattern strategy; identify core of pattern to create repeats	Model and solve real-life addition/ subtraction problems; model and write problems in horizontal/ vertical form; model and identify odd/ even numbers; understand related elements of a fact	Make real-life connection using real objects and group counting patterns; rewrite horizontal problems to vertical format	Model patterns using counters and pattern blocks; explain how to demonstrate pattern using number line and counters; write to demonstrate understanding of pattern	Use a basic fact and a tens counting pattern to find products; use predict and test strategy to identify pattern; use draw a picture or make a table strategy to solve problems

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)
CLUSTER V: PATTERNS AND ALGEBRA

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
MACRO (C): Recognize and describe change in quantities					
KNOWLEDGE: The student should be able to:					
1. Recognize that certain quantities (such as temperature, bank balance, height) change over time				Use data on line graph to model temperature collected over time; record and chart temperature during a day, week, month, etc.	Analyze line graph to tell where increases and decreases occur; use graph data to draw conclusions concerning events
PROBLEM SOLVING SKILLS: In problem settings, using abilities that comprise the power base, the student should be able to:					
2. Describe how certain quantities change over time				Interpret data finding recorded on line graph	Describe activity which causes change using graph data: explain degree change shown on thermometer

CONTENT DOMAIN OUTLINE	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4
3. Explain how a chance in one quality can produce a corresponding change in another		Model and describe a function machine for addition and subtraction	Explain skip counting and the pattern it creates; extend addition by skip counting by 10 and 100 using mental math	Use linear table data to identify rule and explain change; draw arrays to show how the number of groups increases the total amount	Explain why rule on frequency chart changes input/ output; explore relationship between area/ perimeter, describe growing line caused by plotting of ordered pairs on line graph

NEW JERSEY ELEMENTARY SCHOOL PROFICIENCY ASSESSMENT (ESPA)

Mathematics Manipulatives Listing: Appropriate Grade Levels and Specific Skill Development

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INTRODUCTION

This checklist is designed to assist Kindergarten through fourth grade teachers in their selection of appropriate mathematics manipulatives, for use in specific skill development for their grade level. The project includes a listing of many commercial and non-commercial items, which are identified in the Macro Skill Development for grades K-4.

This project may be used to identify multiple uses and appropriate grade levels for each of the manipulatives. ESPA skill development may be assisted by the use of constructivist activities, which utilize math manipulatives.

Included in this listing is a checklist of appropriate grade levels. It is hoped that this will serve as a useful tool for Kindergarten through grade 4 teachers in lesson planning which builds skills being assessed on ESPA.

Mathematics Manipulative Listing: Appropriate Grade Levels and Specific Skill Development

1. Pattern Blocks – small blocks in six shapes and colors	K	1	2	3	4
Create and extend linear patterns	✓	✓	✓	✓	✓
Model fractions			✓	✓	✓
Measure linear and area units			✓	✓	✓
Cover surface area				✓	✓
Identify and create plane figures			✓	✓	✓
Tessellations					✓
Model geometry concepts of congruence and symmetry				✓	✓

2. Color Tiles – 1 inch square tiles in four different colors	K	1	2	3	4
Count and sort	✓	✓			
Create and extend linear patterns	✓	✓	✓	✓	✓
Measure units of area and perimeter			✓	✓	✓
Build multiplication arrays				✓	✓
Model division				✓	✓
Use as sampling item for probability			✓	✓	✓
Model symmetry			✓	✓	✓
Construct graphs		✓	✓	✓	✓
Model number facts		✓	✓	✓	
Build and name fractions				✓	✓

3. Geoboards – 7½ inch square plastic pegboards	K	1	2	3	4
Model basic geometric elements: lines, angles, etc.			✓	✓	✓
Build plane geometry figures	✓	✓	✓		
Measure units for area and perimeter				✓	✓
Model fractions		✓	✓		
Model congruency and symmetry		✓	✓		
Model coordinate points on a grid			✓	✓	

4. Linking Cubes – 2 centimeter plastic cubes that connect on all sides	K	1	2	3	4
Display sets	✓	✓	✓		
Model counting of sets	✓	✓			
Build tens, hundreds, etc.		✓	✓	✓	
Model addition and subtraction number sentences		✓	✓		
Model multiplication arrays				✓	✓
Model squaring and cubing of a number				✓	✓
Model units of measure for area and volume				✓	✓
Build 3-dimensional figures		✓	✓	✓	

5. Attribute Blocks – Small plastic pieces in five shapes, three colors and two thickness	K	1	2	3	4
Count and sort	✓	✓	✓		
Investigate nesting patterns		✓	✓		
Model symmetry		✓	✓		
Model fractions			✓	✓	
Sequence and Logic puzzles				✓	✓
Attribute grouping circles with Venn Diagram			✓	✓	✓

6. Centimeter Cubes – 1 centimeter plastic cubes	K	1	2	3	4
Build patterns				✓	✓
Sort and Classify				✓	✓
Model multiplication Arrays				✓	✓
Model division concepts with remainders				✓	✓
Build 3-dimensional figures				✓	✓
Model problem solving				✓	✓

7. Cuisinaire Rods – Color related plastic rods in 10 sizes	K	1	2	3	4
Model operations				✓	✓
Build fractions				✓	✓
Model units of measure				✓	✓
Model decimal fractions				✓	✓
Create tables and graphs				✓	✓

8. Base Tens Blocks – plastic blocks representing ones, tens, hundreds, and thousands	K	1	2	3	4
Model place value			✓	✓	✓
Model number concepts			✓		
Reinforce counting			✓	✓	
Build numbers to use in comparing values			✓	✓	
Model computation with regrouping			✓	✓	

9. Hundred Chart – 12 inch square chart showing numbers 1-100	K	1	2	3	4
Identify counting patterns		✓	✓	✓	
Model addition and subtraction		✓	✓	✓	
Reinforce number concepts		✓	✓	✓	

10. Dominoes – game pieces with 0-6 dots on each side of the face	K	1	2	3	4
Reinforce number concepts	✓	✓			
Model addition and subtraction facts		✓	✓		
Solve for missing addend		✓	✓	✓	
Practice mental computation skills		✓	✓		

11. Wooden Beads and Lace – colorful, hardwood spheres, cubes, and cylinder beads	K	1	2	3	4
Identify 3-dimensional shapes	✓	✓			
Sorting and Counting	✓	✓			
Create and extend patterns	✓	✓			

12. Two-Color Counters – one inch diameter plastic chips with one side red and one side yellow	K	1	2	3	4
Count and sort	✓				
Model addition and subtraction facts		✓	✓		
Name fractions of a group			✓	✓	✓
Use as a sampling item for probability		✓	✓	✓	✓
Solve for missing addend			✓	✓	

13. Color Links	K	1	2	3	4
Create and extend patterns	✓				
Count and sort	✓				
Model non-standard measurement	✓	✓			
Model place value-tens and ones		✓	✓		

14. Tangram – seven piece shape puzzle	K	1	2	3	4
Explore geometric shapes			✓	✓	
Name, compare and transform shapes				✓	✓
Model lines, congruence and symmetry				✓	✓
Develop spatial awareness			✓	✓	✓

15. Pentominoes – 12 puzzle pieces in varied shapes	K	1	2	3	4
Explore properties and relationships of shapes				✓	✓
Model transforming shapes				✓	✓
Explore area and perimeter of shapes				✓	✓
Develop spatial visualization				✓	✓
Model Tessellations					✓

16. Fraction Stackers – interlocking fraction cubes	K	1	2	3	4
Name fractions				✓	✓
Demonstrate fraction relationships				✓	✓
Model decimals and percents					✓
Compare fractions					✓

17. Dice – typical game dice with 0-6 dots on each face	K	1	2	3	4
Reinforce number concepts		✓			
Model addition and subtraction facts		✓	✓		
Solve for missing addends		✓	✓		
Practice mental computation		✓	✓	✓	
Use as a sampling item for probability				✓	✓
Model skip counting and multiplication				✓	✓
Random number generators for creative large numbers				✓	✓

18. Playing Cards – regular deck	K	1	2	3	4
Reinforce number concepts		✓	✓		
Compare number values		✓	✓		
Model addition and subtraction		✓	✓		
Fact mastery		✓	✓		
Solve for missing addends		✓	✓		
Practice mental computation skills			✓	✓	✓

Thesis Chapter 5

This document is perceived to be a viable tool for teacher use. Teachers understand their responsibility of teaching their students and, at the same time, their accountability to increasing test scores. This document is intended as a reference, which teachers may utilize, in order to better fulfill their commitment to their students.

Kelsey (2001) challenges teachers to ask themselves two very important questions each day. First, "Did my students learn anything today ?" and secondly, "Did they learn what I intended for them to learn today?" She further expresses concern in regards to teaching to the test. "Can Standardized testing capture the complexities of genuine educational experiences? I fear not, and fear still that teaching to the test will continue to dominate many instructional hours as we school, rather than educate our youth." (Kelsey 2001)

This document may serve as an important resource, for all elementary teachers, while asking themselves those two very important daily questions. This document may serve as a tool to alleviate some of the "schooling" of our students by presenting test relevant mathematics activities designed to develop mathematics knowledge and understanding.

The "Macro Development and Open-ended Listing For Grade 4" is hoped to be an invaluable tool for fourth grade teachers. The fourth grade teachers carry the immediate burden of preparing the students, not only for the mathematics content of the E.S.P.A., but also for the specific testing format. This section provides the teachers with a detailed description of both the Macro Skills and the format of the open-ended section of the test. Teachers may utilize each of the sample questions and accompanying rubrics in preparation with their students.

By the examination of the scoring rubric, they may gain insights into the state scoring procedure. It is also recommended that teachers share the scoring rubrics with their students, to promote a better understanding, by the students, of the type of responses which are expected on this portion of the E.S.P.A. Teachers may also utilize this section of the thesis to evaluate potential practice questions, to create their own questions or to develop their own scoring rubrics for additional practice questions.

Elementary teachers may utilize the "Macro Skill Development For Grades K-4" in many aspects of mathematics lesson planning and development. Teachers may refer to it during their weekly lesson planning, as each skill area is written in the form of a student outcome with accompanying activities and manipulatives. Teachers may use it to reference activities from their curriculum guide or text book in order to identify which Macro Skill is being developed during a particular lesson or unit of study. They may use it to follow the mathematics content of their particular grade level, in order to see the scope of the Macro Skill development for that grade. They may then look to earlier grade levels to establish student prior knowledge of a particular Macro Skill. They may look back to assess what has already been developed and then look ahead to future grade levels, in order to identify what skills students will be expected to attain in the future.

It appears, to this researcher, that it is important for all elementary teachers to identify where the mathematics skills, which are taught in their grade level, fit into the elementary mathematics scope of the curriculum. By examining this document, each teacher may identify at which grade level and with which constructivist activitiy the development of specific mathematics skills takes place.

They will then better recognize the relevance of every skill being taught to students.

By examining the "Macro Skill Development for Grades K – 4", some interesting patterns where noted concerning the development of the Mathematics Macro Skills. It appears that many skills are introduced in kindergarten, and grow each year through the fourth grade. Other skills are developed in specific grades only, such as grades K-2 or grades 3-4. Many skills have minimal development in grades K-2 and others are developed solely in those grade levels. It is thought by this researcher, that the blocks on the chart serve as "stepping stones" in the development of these skills in our students. It is important for teachers to note which skills are developed in which grades, in order for them to place the appropriate emphasis in their teaching.

The "Mathematics Manipulative Listing" may be utilized in lesson planning, by presenting a variety of skills which may be developed through the use of a specific manipulative. It allows teachers flexibility in their planning, by alternating different manipulatives to present a skill. It is possible to develop one mathematics skill through the use of as many as five manipulatives. Teachers may refer to this listing, to check on appropriate grade level and to ascertain the availability of manipulatives in their building.

It is the hope, of this researcher, that this document will make a major contribution to the increase of student proficiency on the mathematics section of the New Jersey E.S.P.A. That is the most obvious of goals of this project. However, it is also hoped that this project may foster a new level of understanding between elementary teachers, and within all grade levels, as to the content of the State Standards for mathematics and the quality teaching which is necessary in order to complete our educational goals.

Further development of this project may result in additional in-servicing for fourth grade teachers, in order to disseminate this pertinent information. The in-service may also include a component designed to inform principals as to the importance of tracking the development of mathematics skills from kindergarten through fourth grade. This may help in alleviating some of the demand which is placed directly on the fourth grade teachers for the responsibility of E.S.P.A. scores. It may also make preparation for the E.S.P.A. much less difficult by informing all teachers and administrators where our students need to be guided to by the completion of fourth grade.

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